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# Form factors in the "point form" of relativistic quantum mechanics: Single- and two-particle currents

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**Abstract.** Electromagnetic and Lorentz-scalar form factors are calculated for a bound system of two spinless particles exchanging a zero-mass scalar particle. Different approaches are considered including solutions of a Bethe-Salpeter equation, a "point form" approach to relativistic quantum mechanics and a non-relativistic one. The comparison of the Bethe-Salpeter results, which play the role of an "experiment" here, with the ones obtained in "point form" in single-particle approximation, evidences sizable discrepancies, pointing to large contributions from two-body currents in the latter approach. These ones are constructed using two constraints: ensuring current conservation and reproducing the Born amplitude. The two-body currents so obtained are qualitatively very different from standard ones. Quantitatively, they turn out not to be sufficient to remedy all the shortcomings of the "point form" form factors evidenced in impulse approximation.

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## 1 Introduction

The point form of relativistic quantum mechanics is much less known than the instant and front forms [1], which have been extensively used for describing few-body systems. Recently, a calculation of the nucleon form factors in the former approach has revealed to be in surprisingly good agreement with experiment [2]. There are slight discrepancies for the magnetic moments or at the highest momentum transfers considered in the work, around $Q^2 = 3$–$4$ $(\mathrm{GeV}/c)^2$, but in view of the simplicity of the ingredients involved in the calculation, this is at first sight a negligible point.

The examination of the calculation immediately raises questions. It is a well-known phenomenology that the nucleon form factors at low $Q^2$ are largely dominated by the coupling of the photon to the nucleon via $\omega$ and $\rho$ exchanges (vector meson dominance [3]). The calculation of ref. [2] leaves no room for this important physical contribution, and nothing indicates that the corresponding phenomenology is accounted for in a hidden way by relativistic effects incorporated in the formalism. It is also known that effects in relation with the pion cloud of the nucleon explain the neutron charge radius [4,5] and there is no need to invoke relativity in this respect. On the other hand, it is surprising that quark form factors (which may account for the above coupling of the $\omega$ and $\rho$ mesons) are discarded while they are needed in the construction of the quark-quark interaction used to describe the nucleon [6]. An attempt to incorporate the above physics at the quark level was recently made [7].

When looking at a given system in the Breit frame, the parameter that determines the form factor in "point form" is the velocity [8], $v = (Q/2M)/(1 + Q^2/4M^2)^{1/2}$, where $M$ is the total mass of the system. This has surprising consequences. It immediately follows from this expression that the charge squared radius scales like $1/M^2$ and therefore will increase when $M^2 \to 0$. This is opposite to what is generally expected. A smaller mass can be obtained by increasing the attraction between the constituents, with the result that, usually, the radius decreases.

Another way to put the problem is as follows. One can add an arbitrary constant to the interaction without changing the wave function, but changing the total mass. For the same dynamics, one would thus get different form factors, depending on the arbitrary constant added to the Hamiltonian and therefore to the mass of the system under

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consideration. This is a consequence of the formulas used in ref. [2]. We will not discuss in detail the origin of this paradox and how to solve it but its very existence is a fact that cannot be ignored. Notice that the problem arising in the limit $M^2 \to 0$ is not completely academic as it applies to the pion [9]. A simple dimensional argument would lead to a squared radius of the order of $3/m_\pi^2 \simeq 6\,\mathrm{fm}^2$ (!).

Another point concerns current conservation. The calculation made in ref. [2] does not incorporate mesonic exchange currents that the exchange of charged mesons like the pion implies to preserve this property. Independently of this however, it can be checked that current conservation is not fulfilled. This feature is shared by other approaches. The fact that the total momentum in the "point form" approach contains the interaction, which translates into a particular form of momentum conservation, could make the problem more severe than in the other approaches.

The large difference between the relativistic and non-relativistic calculations has been attributed by the authors themselves to boost effects. Curiously, similar effects have not shown up in other approaches. To take into account the Lorentz contraction, it has been proposed to replace the argument of the form factor, $Q^2$, by $Q^2/(1+Q^2/4M^2)$ (see refs. [10,11] for a discussion). This recipe, only valid at small $Q^2$, gives an effect that goes in a direction opposite to that found in ref. [2]. Calculations of the deuteron electro-disintegration near threshold on the light-front, which were physically incomplete but were supposed to account for the various boost effects ensuring the covariance of the results, have evidenced no sizable effect up to $Q^2 = 10$ $(\mathrm{GeV}/c)^2$ [12,13]. This shows that boost effects can be quite small on a large range of momentum transfers in some cases. It is likely that they show up only for some observables like the nucleon or pion form factors. In this case, non-relativistic estimates of the nucleon (pion) form factor are expected to scale like $Q^{-8}$ $(Q^{-4})$ at high momentum transfer for a quark-quark force which behaves like $1/r$ at small distances [14]. The discrepancy with the QCD expectation, $Q^{-4}$ $(Q^{-2})$, is essentially due to a boost effect characterizing spin-(1/2) constituents, which is therefore expected to increase the form factor at high $Q$ [15]. In the "point form" calculation of the nucleon form factor performed up to now, the effect goes the other way round. Of course, it may show up at larger $Q^2$ but this does not seem to be the tendency evidenced by the results. A similar drop-off is observed or expected in other calculations (see for instance ref. [15]). The point is that these calculations miss further contributions such as those of extra components in the light-front wave function or contact terms [16]. In their absence, the nucleon and pion form factors would not have the correct asymptotic behavior.

The above observations, quite puzzling in some cases, have motivated calculations of form factors in a simple theoretical model which could minimize as much as possible uncertainties due to spin or intrinsic form factors of the constituents [17]. This model consists of two distinguishable, spin-less particles interacting by the exchange of a spin-less, zero-mass boson (Wick-Cutkosky model [18, 19]). What accounts for the experiment is a calculation performed using solutions of the Bethe-Salpeter equation [20], which are easily obtained in this case. Form factors for the lowest bound states can be calculated exactly without much effort and the single-particle current used in the calculations ensures current conservation in all cases. Though it is not quite realistic, this model therefore provides a useful testing ground for various relativistic approaches. It was used by Karmanov and Smirnov, for instance, to check the validity of the calculation of the form factors of $l = 0$ and $l = 1$ states in the light-front approach [21]. The form factors calculated in this model will be referred to as "exact" or "experimental" ones in the following. Calculations based on the same "point form" approach as mentioned above were performed using solutions of a mass operator reproducing the spectrum of the Wick-Cutkosky model. Examination of the results so obtained revealed that form factors were missing the "experimental" ones with two respects. The fall-off of form factors is too fast (the power law behavior of the Born amplitude is missed) and the charge radius tends to be too large, especially when the binding energy of the system under consideration increases.

The discrepancy can be ascribed to the inadequacy of the single-particle current to describe the bulk of the form factors, requiring contributions from two-body currents which, in comparison with other approaches, are quite sizable [17]. An alternative would consist in improving the "point form" implementation. The one mostly referred to in recent applications, also considered here, implies hyperplanes perpendicular to the velocity of the system. This feature was foreseen by Sokolov [22], who noticed that this approach is not identical to the one proposed by Dirac, which relies on a hyperboloid surface[1]. Proposals for improvements have been sketched in refs. [23,24].

In the present paper, we concentrate exclusively on the first alternative, namely the introduction of two-body currents which is explicitly done for the first time in the present approach. This choice is motivated by two observations. While the agreement with experiment is rather good for the "point form" proton magnetic form factor, a closer examination clearly evidences the need for two-body currents. This can be seen immediately when considering the ratio $G_M(Q^2)/G_D(Q^2)$, which suddenly drops off around $Q^2 = 3$–$4$ $(\mathrm{GeV}/c)^2$ [25]. On the other hand, the analysis of various calculations of form factors made in the light-front approach shows that large contributions of two-body currents are required as soon as one drops the condition $q^+ = 0$, which has no counterpart in the "point form" formalism [26–28].

Results of a previous work [17] are first extended to a Lorentz-scalar probe as well as to different mass operators,

[1] The name point form was given by Dirac in relation with the fact that the hyperboloid surface is invariant under Lorentz transformations around some point, $x = 0$ for instance. The two approaches have in common that the interaction is only contained in the four generators $P^\mu$. To emphasize the difference with this original approach, we will use the notation with quotation marks: "point form".

to get a better assessment of the problems raised by the comparison of the "point form" results with the "experimental" ones. The sensitivity to the mass operator is also studied. We then consider two-body currents with a double aim: to restore current conservation (in the case of an electromagnetic probe) and to reproduce the Born amplitude. While doing so, we faced a number of questions. Some are specific to the implementation of the "point form" approach referred to here. However, it turns out that other ones have a more general character and also occur in different forms of relativistic quantum mechanics. Solutions that we considered could therefore be useful elsewhere after being appropriately adapted. They have been accounted for in ref. [24], which was motivated by the question of whether features evidenced by the "point form" approach were shared by the instant- and front-form ones. However, these improvements give rise to a more complicated single-particle current, making the derivation of the associated two-body currents more involved. For the present exploratory work on these currents, we will consider the simplest one-body current. As will be seen, the associated two-body currents are already sophisticated enough.

It is not *a priori* certain whether the two-body currents we want to consider will be sufficient. In relativistic quantum mechanics, the constituents of a system have an effective character. The derivation of their interaction and their currents is, apart from some constraints, an open problem. Though good results were obtained for form factors in the instant form (Breit frame) and the front form ($q^+ = 0$) [24], one should keep in mind that the problem of determining currents is not necessarily closed. Recent studies within the light-front approach have revealed a strong frame dependence of individual contributions, pointing to large contributions of two-body currents in some cases [26–28]. Being motivated by Lorentz covariance, it is not clear whether these currents have some relevance here, where this property is intrinsic to the formalism. Moreover, part of these studies were performed within a field-theory approach. However, there is often some correspondence between conclusions reached in such works and those obtained in relativistic quantum mechanics.

While studying the contribution of two-body currents, we have especially in mind the pion form factor, whose recent re-evaluation [9], partly anticipated by a remark made above, evidences considerable disagreement with experiment. The new estimate now leaves room for well-known contributions in relation with the Goldstone nature of the pion [29]. The discrepancy could have many sources in relation with this property, with the spin-(1/2) nature of the constituents or the implementation of the "point form" approach itself. As far as we can see, the last ingredient is the most sensitive one. The present work, which aims to settle the implementation of this approach in all aspects, can therefore provide a relevant information as for the description of the pion form factor. It is not clear whether, ultimately, this program will be of practical interest and will compete with a field-theory approach [29]. We nevertheless believe that it is part of a sound scientific attitude to understand the gross features pertinent to the description of the properties of few-body systems in either approach.

The plan of the paper is as follows. The second section is devoted to reminding the impulse approximation expressions of the form factors we are calculating and also includes the case of a scalar probe. In the third section, we extend numerical results presented in a previous paper [17] to the scalar form factor and to other mass operators. The fourth section is concerned with general comments inspired by the results so obtained. It deals with both the low momentum transfers, where current conservation is an important constraint, and high momentum ones, where the consideration of the Born amplitude provides an important benchmark. The derivation of two-body currents that allow one to fulfill these two constraints is made in the fifth section. This is done consistently with the one-body current that is used. Their contributions to form factors are then given. Results involving both contributions are presented at the end of the section, together with a discussion of that part due to two-body currents. Many expressions pertinent to the present work are gathered in the appendix.

## 2 Form factors in different formalisms

Extending our previous work [17], we here consider form factors relative to both a scalar and an electromagnetic probe. Quite generally, the corresponding matrix element between two states with $l = 0$, possibly different, may be expressed as

$$\sqrt{2E_f 2E_i}\ \langle f|J^\mu|i\rangle = F_1(q^2)(P_f^\mu + P_i^\mu) + F_2(q^2)q^\mu,$$
$$\sqrt{2E_f 2E_i}\ \langle f|S|i\rangle = F_0(q^2)\,(4m), \tag{1}$$

where $q^\mu = P_f^\mu - P_i^\mu$ and $q^2 = -Q^2$. The operators $S$ and $J^\mu$ on the l.h.s. of eq. (1), describe the interaction with the external probe, respectively of Lorentz-scalar and vector types. Due to current conservation, the form factors $F_1(q^2)$ and $F_2(q^2)$ have to fulfill the following relationship:

$$F_1(q^2)\,(M_f^2 - M_i^2) + F_2(q^2)\,q^2 = 0. \tag{2}$$

For an elastic process, this is automatically fulfilled since $F_2(q^2)$ vanishes identically from symmetry arguments alone, but this does not imply that current conservation holds at the operator level, as it should. For an inelastic process, eq. (2) implies that $F_1(q^2) \to 0$ with $q^2$.

The normalization of the form factors in eq. (1) is for some part arbitrary. Assuming that the system under consideration is made of one charged and one neutral particle, it is appropriate to normalize $F_1(q^2)$ such that $F_1(q^2 = 0) = 1$. In absence of a conservation law for the scalar probe, we normalize the scalar form factor such that $F_0(q^2)$ and $F_1(q^2)$ coincide in the non-relativistic limit. Even so, other normalization factors with the same non-relativistic limit in eq. (1) could be chosen, such as $2M$ instead of $4m$, but we did not find any compelling reason to do it (see discussion in sect. 3).

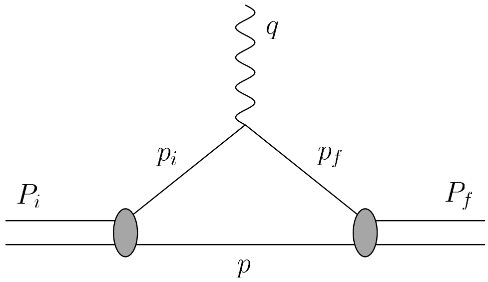


**Fig. 1.** Representation of a scalar particle or virtual photon absorption on a two-body system, with indication of the kinematical definitions.

In the following, we successively consider form factors in the Wick-Cutkosky model and in the "point form" approach. Some of the matter given in an earlier paper [17] is provided in appendices A and B together with a few formulas. The contribution we intend to calculate is shown in fig. 1.

### 2.1 Form factors in the Wick-Cutkosky model

#### 2.1.1 Interaction and Bethe-Salpeter amplitudes

What will account for our "experiment" is based on the Wick-Cutkosky model. The Bethe-Salpeter amplitudes take in this case a relatively simple integral form for the lowest state with a given angular momentum $l$:

$$\chi_P(p) = \int \mathrm{d}z \, \frac{g_n(z)\, \mathcal{Y}_l^m(\vec{p})}{(m^2 - \frac{1}{4}P^2 - p^2 - z\, P{\cdot}p - i\epsilon)^{n+2}}, \quad (3)$$

with $n = l + 1$. For the first radial excitation, the Bethe-Salpeter amplitude reads

$$\chi_P(p) = \int \mathrm{d}z \, \frac{g_n(z)\,(p^2 + m^2 - \frac{1}{4}P^2)\, \mathcal{Y}_l^m(\vec{p})}{(m^2 - \frac{1}{4}P^2 - p^2 - z\, P{\cdot}p - i\epsilon)^{n+2}}, \quad (4)$$

where now $n = l + 2$. In these expressions, $\mathcal{Y}_l^m(\vec{p}) = |\vec{p}|^l Y_l^m(\hat{p})$ and $g_n(z)$ is a solution of the second-order differential equation [18,19]:

$$(1 - z^2)\, g_n''(z) + 2(n-1)\, z\, g_n'(z) - n(n-1)\, g_n(z) + \frac{\alpha}{\pi(\epsilon^2 z^2 + 1 - \epsilon^2)}\, g_n(z) = 0\,, \quad (5)$$

with $\epsilon^2 = P^2/(4m^2)$ and the boundary conditions $g_n(z = \pm 1) = 0$. Only normal solutions (without node) are considered here. In the small-binding limit, the function $g_1(z)$ of the ground state is given by $1 - |z|$, while in the deep-binding limit ($M = 0$), $g_1(z) \propto 1 - z^2$.

#### 2.1.2 Expressions of form factors

For the model under consideration here, the general (and exact) expression of the matrix element of the current can be written in terms of the Bethe-Salpeter amplitudes:

$$\sqrt{2E_f\, 2E_i}\; \langle f|S|i\rangle = i \int \frac{\mathrm{d}^4 p}{(2\pi)^4}\, (2m) \times \chi_{P_f}\left(\frac{1}{2}P_f - p\right) (p^2 - m^2)\, \chi_{P_i}\left(\frac{1}{2}P_i - p\right),$$

$$\sqrt{2E_f\, 2E_i}\; \langle f|J^\mu|i\rangle = i \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \left(P_f^\mu + P_i^\mu - 2\, p^\mu\right) \times \chi_{P_f}\left(\frac{1}{2}P_f - p\right) (p^2 - m^2)\; \chi_{P_i}\left(\frac{1}{2}P_i - p\right). \quad (6)$$

According to our normalization convention, a factor $2\,m$ has been introduced in the above expression of the scalar form factor. In a scalar theory, like the Wick-Cutkosky model, this factor is often separated out to make the coupling constant dimensionless and directly comparable to $\alpha_{\rm QED}$.

Using eqs. (3) and (4), the calculation of the matrix element, eq. (1), can be partly performed by employing the Feynman method. A few expressions of interest here are given in appendix A for both elastic and inelastic cases. Though the calculation is not straightforward, it can be checked that the current conservation, eq. (2), is verified in the inelastic case.

### 2.2 Form factors in the "point form" approach

The "point form" approach to relativistic quantum mechanics is characterized by the property that, among the 10 generators of the Poincaré group, only the four momenta $P^\mu$ contain the interaction. These ones can be written as the sum of the free particle and interaction contributions:

$$P^\mu = P^\mu_{\rm free} + P^\mu_{\rm int} = M\, V^\mu\,, \quad (7)$$

where the last equality defines the four-velocity operator $V^\mu$ in terms of the mass operator $M = \sqrt{P^2}$.

In the implementation of the "point form" employed in recent applications, the simplest choice compatible with the Poincaré algebra has been made for the interaction part of the four-momentum. This one assumes the form

$$P^\mu_{\rm int} = M_{\rm int}\, V^\mu, \quad (8)$$

which, together with eq. (7), implies

$$V^\mu\, (M - M_{\rm int}) = V^\mu_{\rm free}\, M_{\rm free}, \quad (9)$$

where $M_{\rm free} = \sqrt{P^2_{\rm free}}$. From this one obtains

$$V^\mu = V^\mu_{\rm free}, \qquad M = M_{\rm free} + M_{\rm int}. \quad (10)$$

The last relation, which could also be used as a definition of $M_{\rm int}$, is consistent with the choice of eq. (8).

The form of the interaction part of the four-momentum considered in eq. (8) can be associated to a physics description on a hyper-plane perpendicular to the four-velocity of the system, an observation made previously by Sokolov [22]. As for the interaction term $M_{\rm int}$, it may be chosen according to some theoretical prejudice or to reproduce some experimental spectrum as done in ref. [2]. Instead of $M$, one can also use the square of the operator [8]. This has some advantage since, in the two-body case, $M^2$ is very close to a Schrödinger equation. The solutions of this one may therefore also be used as was done in ref. [17]. In this case, it turns out that the theoretical spectrum obtained with a Coulomb-like potential reproduces rather well the one of the Wick-Cutkosky model. We have thus a set of analytic wave functions that can be used for the calculation of form factors. However, while doing this, one has to worry that the currents associated to the mass operators $M^2$ and $M$ may not be the same (a conserved current is more easily built in one case than in the other).

### 2.2.1 Mass operator and solutions

In this work, we will still refer to the above calculations made with the Coulomb-like potential (denoted model v0, see appendix B) but we will also consider solutions of a linear mass operator. Consistently with eq. (10), the corresponding equation takes the form

$$M\,\psi(k) = 2\,e_k\,\psi(k) + \int \frac{\mathrm{d}\vec{k}\,'}{(2\pi)^3}\,V_{\rm int}(\vec{k},\vec{k}\,')\,\psi(k'), \qquad (11)$$

where $e_k = \sqrt{m^2+k^2}$, while $\vec{k}$ represents an internal variable that, in the non-relativistic limit, could be identified with the relative momentum. Without certitude about which approach is the best, this will give insight on the related uncertainty. The main difficulty is to derive an interaction to be used in eq. (11) such that it reproduces the spectrum of the Wick-Cutkosky model. The first-order term one can think of is motivated by a standard field-theory approach to the derivation of a one-boson interaction. It is given by

$$V_{\rm int}(\vec{k},\vec{k}\,') = -\frac{m}{e_k}\,\frac{g^2}{(\vec{k}-\vec{k}\,')^2}\,\frac{m}{e_{k'}}\,. \qquad (12)$$

This model, denoted v1, which is non-local, misses however properties of the Wick-Cutkosky model that were reproduced by the Coulomb-like potential, such as the degeneracy of $1p$ and $2s$ states. Relying on the fact that the square of the mass operator should be close to the one which works ($(2e_k + M_{\rm int})^2 = 4e_k^2 - 4m\frac{\alpha_{\rm eff}}{r}$ in configuration space), an extra term can be derived:

$$\Delta V_{\rm int}(\vec{k},\vec{k}\,') = -\frac{m}{e_k}\left(\frac{g^2}{(\vec{k}-\vec{k}\,')^2}\,\frac{2\,e_k\,e_{k'} - m\,(e_k+e_{k'})}{m\,(e_k+e_{k'})} + \frac{g^4}{32\,m\,|k-k'|}\,\frac{8\,m^3}{(e_k+e_{k'})(e_k+m)(e_{k'}+m)}\right)\frac{m}{e_{k'}}\,. \qquad (13)$$

The expression is exact for the part linear in $g^2$ and includes corrections at the order $g^4$ in an approximate way (exact in the lowest $1/m$ order with correct asymptotic $1/k^4$ power law (up to log terms)). The addition of the above correction to the interaction given in eq. (12) defines an improved model, denoted v2.

### 2.2.2 Expressions of form factors

Solutions of the mass operator $M$ can now be used for the calculation of form factors. This was described in ref. [8] for the matrix element of the single-particle current. However, instead of using expressions where appropriate boosts have to be performed, we rely on expressions whose Lorentz covariance is explicit:

$$\begin{aligned}\sqrt{2E_f\,2E_i}\;\langle f|S|i\rangle &= \sqrt{2M_f\,2M_i}\,\frac{1}{(2\pi)^3}\\ &\times\int \mathrm{d}^4p\,\mathrm{d}^4p_f\,\mathrm{d}^4p_i\,\mathrm{d}\eta_f\,\mathrm{d}\eta_i\,\sqrt{(p_f+p)^2\,(p_i+p)^2}\\ &\times\delta(p^2-m^2)\,\delta(p_f^2-m^2)\,\delta(p_i^2-m^2)\,\theta(\lambda_f\cdot p_f)\,\theta(\lambda_f\cdot p)\\ &\times\theta(\lambda_i\cdot p)\,\theta(\lambda_i\cdot p_i)\,\delta^4(p_f+p-\lambda_f\eta_f)\,\delta^4(p_i+p-\lambda_i\eta_i)\\ &\times\phi_f\left(\left(\frac{p_f-p}{2}\right)^2\right)\phi_i\left(\left(\frac{p_i-p}{2}\right)^2\right)(2m), \qquad (14)\end{aligned}$$

$$\begin{aligned}\sqrt{2E_f\,2E_i}\;\langle f|J^\mu|i\rangle &= \sqrt{2M_f\,2M_i}\,\frac{1}{(2\pi)^3}\\ &\times\int \mathrm{d}^4p\,\mathrm{d}^4p_f\,\mathrm{d}^4p_i\,\mathrm{d}\eta_f\,\mathrm{d}\eta_i\,\sqrt{(p_f+p)^2\,(p_i+p)^2}\\ &\times\delta(p^2-m^2)\,\delta(p_f^2-m^2)\,\delta(p_i^2-m^2)\,\theta(\lambda_f\cdot p_f)\,\theta(\lambda_f\cdot p)\\ &\times\theta(\lambda_i\cdot p)\,\theta(\lambda_i\cdot p_i)\delta^4(p_f+p-\lambda_f\eta_f)\,\delta^4(p_i+p-\lambda_i\eta_i)\\ &\times\phi_f\left(\left(\frac{p_f-p}{2}\right)^2\right)\phi_i\left(\left(\frac{p_i-p}{2}\right)^2\right)(p_f^\mu+p_i^\mu), \qquad (15)\end{aligned}$$

where $\lambda^\mu_{i,f}$ are unit four-vectors proportional to the four-momenta of the total system in the initial and final states, $\lambda_i^\mu = P_i^\mu/M_i$ and $\lambda_f^\mu = P_f^\mu/M_f$. These four-vectors can be expressed in terms of the corresponding velocities, $\lambda^0 = (\sqrt{1-v^2}\,)^{-1}$ and $\vec{\lambda} = \vec{v}\,(\sqrt{1-v^2}\,)^{-1}$.

Except obviously for the current that behaves like a four-vector, all quantities in the above expressions are Lorentz invariant. This is achieved by the introduction of auxiliary variables $\eta_i$ and $\eta_f$, which play the role of an off-energy shell invariant mass. When they are integrated over, they give rise to the following three-dimensional $\delta$ functions:

$$\delta\!\left(\vec{p}_i+\vec{p}-\frac{\vec{\lambda}_i}{\lambda_i^0}(p_i^0+p^0)\right) \text{ and } \delta\!\left(\vec{p}_f+\vec{p}-\frac{\vec{\lambda}_f}{\lambda_f^0}(p_f^0+p^0)\right). \qquad (16)$$

These relations are pertinent to the "point form" approach referred to throughout this paper and account for the fact that the velocity $\vec{v}$, defined as the ratio of the sum of the momenta $\sum\vec{p}_j$ and the sum of the kinetic energies $\sum e_j$,

**Table 1.** Elastic form factor $F_1(q^2)$ for the ground state: non-relativistic (N.R.) and "point form" (P.F.) calculations are performed with Coulombian wave functions (model v0, see appendix B). The binding energies of the states, in units of the constituent mass $m$, are given by $E = 0.0842$ ($\alpha = 1$), $E = 0.432$ ($\alpha = 3$) and $E = 2.0$ ($\alpha = 2\pi$). In the last case, results for two slightly different values of $E$ are given for the "point form" results (see explanation in the text).

| $Q^2/m^2$ | 0.01 | 0.1 | 1.0 | 10.0 | 100.0 |
|---|---|---|---|---|---|
| $\alpha = 1$ | | | | | |
| B.S. | 0.984 | 0.856 | 0.309 | 0.137-01 | 0.213-03 |
| N.R. | 0.985 | 0.864 | 0.323 | 0.136-01 | 0.169-03 |
| P.F. | 0.984 | 0.853 | 0.299 | 0.974-02 | 0.343-04 |
| $\alpha = 3$ | | | | | |
| B.S. | 0.996 | 0.962 | 0.705 | 0.139 | 0.503-02 |
| N.R. | 0.997 | 0.968 | 0.740 | 0.146 | 0.338-02 |
| P.F. | 0.995 | 0.949 | 0.621 | 0.563-01 | 0.228-03 |
| $\alpha = 2\pi$ | | | | | |
| B.S. | 0.998 | 0.983 | 0.848 | 0.339 | 0.285-01 |
| N.R. | 0.999 | 0.988 | 0.886 | 0.379 | 0.190-01 |
| P.F.$_{(E=1.90)}$ | 0.614 | 0.398-01 | 0.111-03 | 0.126-06 | 0.127-09 |
| P.F.$_{(E=1.95)}$ | 0.187 | 0.143-02 | 0.193-05 | 0.199-08 | 0.200-11 |

is conserved for a given system [8]. They replace the conservation of momenta in the instant-form approach. It has not been possible to show that they strictly follow from describing physics on a hyperboloid surface [30]. Instead, they can be obtained when this surface is taken as a hyperplane orthogonal to the four-velocity of the system, consistently with the form of eq. (8) and the observation made by Sokolov [22]. On the other hand, it can be checked that, in the c.m., the wave function $\phi$ only depends on the relative momentum of the two particles. Moreover, by direct integration or after performing a change of variable, one recovers that the current of a given system is given by $(\langle J^0\rangle, \langle\vec{J}\rangle) = (1, \vec{v})$, in agreement with the standard normalization of the wave function, $\int \frac{\mathrm{d}\vec{k}}{(2\pi)^3}\,\phi^2(\vec{k}) = 1$. We will come back to this normalization in sect. 5, when considering two-body currents.

The form factors we are interested in, $F_0(q^2)$, $F_1(q^2)$ and $F_2(q^2)$, can be calculated from eqs. (14), (15) in any frame. However, they take a simpler expression in the Breit frame, defined by $\vec{v} = \vec{v}_f = -\vec{v}_i$, with $v$ expressed in terms of the momentum transfer $Q$: $v^2 = \frac{Q^2+(M_f-M_i)^2}{Q^2+(M_f+M_i)^2}$. The electromagnetic form factors are more appropriately expressed in terms of auxiliary quantities, $\tilde{F}_1(q^2)$ and $\tilde{F}_2(q^2)$, which involve the time and spatial parts of the current, respectively. We thus have:

$$F_0(q^2) = \frac{\sqrt{M_f\,M_i}}{2\,m}\int\frac{\mathrm{d}\vec{p}}{(2\pi)^3}\,\phi_f(\vec{p}_{tf})\,\frac{m}{e_p}\,\phi_i(\vec{p}_{ti}),$$

$$\tilde{F}_1(q^2) = \frac{1+v^2}{\sqrt{1-v^2}}\int\frac{\mathrm{d}\vec{p}}{(2\pi)^3}\,\phi_f(\vec{p}_{tf})\,\phi_i(\vec{p}_{ti}),$$

$$\tilde{F}_2(q^2)\,\vec{v} = -\frac{1+v^2}{\sqrt{1-v^2}}\int\frac{\mathrm{d}\vec{p}}{(2\pi)^3}\phi_f(\vec{p}_{tf})\frac{\vec{p}}{e_p}\phi_i(\vec{p}_{ti}), \qquad (17)$$

together with

$$F_1(q^2)\,\sqrt{2M_f\,2M_i} = \tilde{F}_1(q^2)\,(M_f+M_i) - \tilde{F}_2(q^2)\,(M_f-M_i)\,,$$
$$F_2(q^2)\,\sqrt{2M_f\,2M_i} = -\tilde{F}_1(q^2)\,(M_f-M_i) + \tilde{F}_2(q^2)\,(M_f+M_i)\,. \qquad (18)$$

The (Lorentz-) transformed momenta are defined as $(p^x, p^y, p^z)_{ti,f} = (p^x, p^y, \frac{p^z \pm v\,e_p}{\sqrt{1-v^2}})$, together with $e_p = \sqrt{m^2+\vec{p}^2}$.

Contrary to eq. (6), there is no guarantee that current conservation, eq. (2), is fulfilled by eqs. (17), (18). How much it is violated for inelastic transition is of interest.

### 2.3 Non-relativistic form factors

Finally, we recall the non-relativistic expressions of the elastic and inelastic form factors $F_0(q^2)$, $F_1(q^2)$ and $F_2(q^2)$, that can be calculated with the same wave functions as used in eqs. (17). For a local interaction model, like v0, where the simplest single-particle current is conserved, they read

$$F_0(q^2) = F_1(q^2) = \int\frac{\mathrm{d}\vec{p}}{(2\pi)^3}\,\phi_f\left(\vec{p}-\frac{1}{4}\vec{q}\right)\,\phi_i\left(\vec{p}+\frac{1}{4}\vec{q}\right),$$

$$F_2(q^2)\,\frac{\vec{q}}{4m} = -\int\frac{\mathrm{d}\vec{p}}{(2\pi)^3}\phi_f\left(\vec{p}-\frac{\vec{q}}{4}\right)\frac{\vec{p}}{m}\phi_i\left(\vec{p}+\frac{\vec{q}}{4}\right). \quad (19)$$

In this case, it can be checked that the form factors verify the current conservation condition, eq. (2).

In a few cases, form factors involving Coulombian wave functions can be calculated analytically. Their expression is given in appendix B. For a non-local interaction model like v1, which includes a semi-relativistic kinetic energy or normalization factors $m/e$, the expression of the form

**Table 2.** Elastic form factor $F_0(q^2)$ for the ground state: same as in table 1.

| $Q^2/m^2$ | 0.01 | 0.1 | 1.0 | 10.0 | 100.0 |
|---|---|---|---|---|---|
| $\alpha = 1$ | | | | | |
| B.S. | 1.024 | 0.887 | 0.313 | 0.128-01 | 0.18-03 |
| N.R. | 0.985 | 0.864 | 0.323 | 0.135-01 | 0.17-03 |
| P.F. | 0.949 | 0.813 | 0.256 | 0.43-02 | 0.33-05 |
| $\alpha = 3$ | | | | | |
| B.S. | 1.123 | 1.080 | 0.767 | 0.132 | 0.394-02 |
| N.R. | 0.996 | 0.968 | 0.740 | 0.145 | 0.337-02 |
| P.F. | 0.682 | 0.641 | 0.366 | 0.16-01 | 0.15-04 |
| $\alpha = 2\pi$ | | | | | |
| B.S. | 1.247 | 1.222 | 1.016 | 0.338 | 0.217-01 |
| N.R. | 0.997 | 0.987 | 0.885 | 0.378 | 0.189-01 |
| P.F.$_{(E=1.90)}$ | 0.175-01 | 0.435-03 | 0.28-06 | 0.54-10 | 0.79-14 |
| P.F.$_{(E=1.95)}$ | 0.162-02 | 0.335-05 | 0.86-09 | 0.14-12 | 0.18-16 |

**Table 3.** Inelastic form factors $F_0(q^2)$, $F_1(q^2)$ and $F_2(q^2)$, for a transition from the ground state to the first radially excited one: non-relativistic and "point form" results are obtained with Coulombian wave functions (model v0), as in tables 1 and 2. The results correspond to $\alpha = 3$ ($E_i = 0.4322\,m$, $E_f = 0.1036\,m$ for B.S. and $0.098\,m$ for N.R. and P.F.).

| $Q^2/m^2$ | 0.01 | 0.1 | 1.0 | 10.0 | 100.0 |
|---|---|---|---|---|---|
| B.S. | | | | | |
| $F_0$ | 0.538-01 | 0.781-01 | 0.172-00 | 0.537-01 | 0.163-02 |
| $F_1$ | 0.032-01 | 0.298-01 | 0.145-00 | 0.584-01 | 0.214-02 |
| $F_2$ | 0.369-00 | 0.340-00 | 0.165-00 | 0.665-02 | 0.217-04 |
| N.R. | | | | | |
| $F_0$ | 0.032-01 | 0.296-01 | 0.151-00 | 0.550-01 | 0.121-02 |
| $F_1$ | 0.032-01 | 0.296-01 | 0.151-00 | 0.550-01 | 0.121-02 |
| $F_2$ | 0.369-00 | 0.342-00 | 0.174-00 | 0.636-02 | 0.140-04 |
| P.F. | | | | | |
| $F_0$ | −0.046-01 | 0.171-01 | 0.090-00 | 0.099-01 | 0.119-04 |
| $F_1$ | 0.101-01 | 0.372-01 | 0.140-00 | 0.283-01 | 0.139-03 |
| $F_2$ | 0.324-00 | 0.293-00 | 0.119-00 | 0.217-03 | −0.118-04 |

factors may involve slightly different single-particle operators while preserving the Galilean invariance. These ones, which are model dependent, will be given later on together with the two-body currents that are then necessary to fulfill current conservation. These form factors will serve as a useful benchmark for comparison with the "point form" results, which should represent an improvement with respect to the "exact" ones.

## 3 Results in impulse approximation

In this section, we complete results obtained in an earlier paper for electromagnetic form factors [17] by providing scalar ones. They should allow one to get a better insight on how the "point form" approach does with respect to the other ones. Results corresponding to different mass operators are also presented.

Results are presented successively in three tables: for the elastic form factor, $F_1(q^2)$ (table 1), for the elastic form factor, $F_0(q^2)$ (table 2), and for an inelastic transition from the ground to the first radially excited state (table 3). In the two first cases, three values of the coupling constant have been considered: $\alpha = 1$, $\alpha = 3$ and $\alpha = 2\pi$, where $\alpha$ is related to the coupling constant $g^2$ used in eq. (12) by $\alpha = g^2/(4\,\pi)$. These values correspond to a small, a moderate and a large binding energy (4%, 20% and 100% of the total mass of the constituents), respectively. The last value is an extreme one since the total mass is zero but, as sometimes happens, such cases better reveal features pertinent to some approach. In the zero-mass case, results for the "point form" approach essentially vanish at $Q^2 \neq 0$ ($F_1$) or even identically ($F_0$). For this reason, the corresponding results are given for two values of the binding energy, $E = 1.90\,m$ and $E = 1.95\,m$, which allow one to approach the limit $M = 0$, while at the same time values obtained with the Bethe-Salpeter or non-relativistic approaches are essentially unchanged. These results can provide qualitative information on a system like the pion whose total mass is much smaller than the sum of the masses of the constituents. For the inelastic transition, results are presented for the three form factors $F_0(q^2)$, $F_1(q^2)$ and $F_2(q^2)$, and for one value of the coupling constant, $\alpha = 3$.

### 3.1 Elastic charge form factors

Results for the elastic form factor, $F_1(q^2)$ (table 1), have been already discussed in ref. [17]. As noticed there, the non-relativistic calculation agrees relatively well with the "exact" results. Reproducing at the same time the low-momentum range, constrained by the charge associated to the conserved current, and the high-momentum range, constrained by the Born amplitude, the form factor $F_1(q^2)$, calculated in the non-relativistic approach, cannot be wrong by a large amount (up to log terms). As noticed in ref. [17], the "point form" approach departs from the "exact" result at the highest values of $Q^2$ that were considered. Results at $Q^2 = 100\, m^2$ clearly show the failure of this approach in the impulse approximation for a large coupling ($\alpha = 2\pi$), but also at small couplings. This is obviously due to the asymptotic behavior of the form factor that varies like $1/Q^6$ instead of $1/Q^4$ for the "exact" and non-relativistic calculations. As for the tendency of the "exact" results to depart from the non-relativistic ones at these high $Q^2$ values, it signs the onset of log term corrections in the former ones.

### 3.2 Elastic scalar form factors

Results for the scalar form factor $F_0(q^2)$ (table 2) confirm the overall agreement of the non-relativistic calculation with the "exact" one. Contrary to $F_1(q^2)$, some discrepancy appears at $Q^2 = 0$. In absence of a conserved charge in this case, this points to the role of relativistic corrections at low $Q^2$. These ones remain moderate however, including the extreme case $\alpha = 2\pi$ ($M = 0$). In comparison, for the "point form" results the discrepancy with the "exact" ones is larger than for $F_1(q^2)$, both at small and large $Q^2$. At low $Q^2$, part of the effect is due to the factor $M/(2m)$ appearing in eq. (17), which has no counterpart in the other approaches. This effect becomes especially large when approaching the limit $M = 0$, a result which is independent of the way the scalar form factor is defined in eq. (1). The ratio of the $F_0(q^2)$ and $F_1(q^2)$ form factors however depends on this definition, requiring some caution about the conclusion that their comparison can suggest. As there is a close relationship between these form factors in the Wick-Cutkosky model, both numerically and algebraically[2], we are rather tempted to think that, in the "point form" approach, the form factor $F_0(q^2)$ is strongly suppressed with respect to $F_1(q^2)$ at low $Q^2$ in the limit $M \to 0$. At high $Q^2$, the form factor rather scales like $1/Q^8$, instead of $1/Q^4$, hence a larger discrepancy with the other results than for $F_1(q^2)$.

### 3.3 Inelastic form factors

When making a comparison with the "exact" results, it was noticed in ref. [17] that, for the "point form" results, a relative change in sign of the form factors $F_1(q^2)$ and $F_2(q^2)$ occurs, preventing from fulfilling the current conservation constraint given by eq. (2). The fact that the non-relativistic calculation does better than the "point form" one is confirmed by results for the scalar form factor, $F_0(q^2)$. In particular, at low $Q^2$ the first one has the right sign while the other one has not. However, the discrepancy in size is large in both cases. Much better than the elastic case, the present results evidence the role of relativistic corrections. Anticipating on the next section, let us mention that a non-zero value of $F_0(q^2)$ at $Q^2 = 0$ can be obtained by adding in the non-relativistic approach exchange currents (pair term). At high $Q^2$, many statements made for the elastic case could be repeated here. They concern the ratio $F_0/F_1$, the comparison with non-relativistic calculations and the fall-off of the form factors in the "point form" approach.

### 3.4 Other mass operators

Results presented in tables 1-3 have been obtained with a wave function issued from a Coulomb-type potential (model v0). An important question is whether qualitative results obtained so far extend to other interaction models. For comparison, we used a wave function issued from a linear mass operator, eq. (11), with an interaction given by eq. (12). In this calculation, (model v1), the coupling constant has been adjusted to reproduce the same binding energy as the one obtained with the Bethe-Salpeter equation and $\alpha = 3$, giving $\alpha(\mathrm{v1}) = 1.775$. The difference in the couplings is simply due to the fact that the Bethe-Salpeter approach accounts for retardation effects which effectively decrease the strength of the interaction [31,32]. The model v1 does not reproduce the spectrum of the Wick-Cutkosky model as well as the model v0. The binding energy of the first radial excitation is $0.1320\, m$ instead of $0.1036\, m$. Therefore, a comparison of form factors from this model with the exact ones is less instructive. However, a comparison with a non-relativistic–type calculation, using eqs. (19), may still be useful.

No major qualitative difference with previous results is seen at small as well as high $Q^2$ (see table 4). Quantitative differences can be traced back to the interaction model itself. It provides a slightly more rapid decrease of the wave function in momentum space (roughly given by an extra factor $(e_k + m)/(2e_k)$ for small binding energy). This is a consequence of the semi-relativistic kinematics in eq. (10) together with normalization factors in eq. (12). As a result, the wave function at the origin, $\psi_r(0)$, is smaller (see sect. 4 for the role of this quantity).

As a side remark, we notice that the asymptotic values for the form factors in the Coulombian model, v0, and the model v1 have not yet been reached at momentum transfers as large as $Q^2/m^2 = 100$. For v1, the value is too large by $\sim 15\%$ ( $(Q^4/m^4)\, F(Q^2)\big|_{Q^2/m^2=100} \simeq 7.7$ instead of 6.7 asymptotically), while for v0 it is too low by $\sim 10\%$ ( $(Q^4/m^4)\, F(Q^2)\big|_{Q^2/m^2=100} \simeq 34$ instead of 38). The ratio of the asymptotic values, 0.18, is mainly due to the difference in the values of the wave functions at the origin (0.24), of the factor $(e_k + m)/(2e_k)$ at large $k$ (0.5) and of the coupling constants (1.4).

The origin of the above quantitative differences can be checked by using a model more in the spirit of the mass operator of eq. (10), like the one incorporating corrections to the interaction as given by eq. (13). In this model, denoted v2, the coupling is again fitted to the binding energy obtained with the Bethe-Salpeter equation, giving $\alpha(\mathrm{v2}) = 1.327$, which is quite close to the one for the

[2] The two form factors $F_0(q^2)$ and $F_1(q^2)$ are equal in the Born approximation and from higher orders, one expects log corrections leading to $F_1(q^2) = 2\, F_0(q^2)$ in the ultra-relativistic domain.

**Table 4.** Elastic form factors $F_0(q^2)$ and $F_1(q^2)$, calculated with wave functions issued from the interaction models v1, given by eq. (12), and v2, which includes the correction eq. (13). The couplings, $\alpha(\text{v1}) = 1.775$ and $\alpha(\text{v2}) = 1.327$, have been determined to reproduce the binding energy $E = 0.4322\,m$ of the Bethe-Salpeter equation ($\alpha = 3$). Results for the model v0 ($\alpha(\text{v0}) = 1.241$) and B.S. are recalled for comparison.

| $Q^2/m^2$ | 0.01 | 0.1 | 1.0 | 10.0 | 100.0 |
|---|---|---|---|---|---|
| N.R. (v1) | | | | | |
| $F_0 = F_1$ | 0.995 | 0.953 | 0.640 | 0.665-01 | 0.774-03 |
| P.F. (v1) | | | | | |
| $F_0$ | 0.710 | 0.651 | 0.303 | 0.596-02 | 0.281-05 |
| $F_1$ | 0.992 | 0.924 | 0.493 | 0.205-01 | 0.464-04 |
| N.R. (v2) | | | | | |
| $F_0 = F_1$ | 0.996 | 0.966 | 0.727 | 0.132 | 0.279-02 |
| P.F. (v2) | | | | | |
| $F_0$ | 0.689 | 0.645 | 0.358 | 0.140-01 | 0.117-04 |
| $F_1$ | 0.994 | 0.946 | 0.603 | 0.494-01 | 0.177-03 |
| N.R. (v0) | | | | | |
| $F_0 = F_1$ | 0.997 | 0.968 | 0.740 | 0.146 | 0.338-02 |
| P.F. (v0) | | | | | |
| $F_0$ | 0.682 | 0.641 | 0.366 | 0.16-01 | 0.15-04 |
| $F_1$ | 0.995 | 0.949 | 0.621 | 0.56-01 | 0.23-03 |
| B.S. | | | | | |
| $F_0$ | 1.123 | 1.080 | 0.767 | 0.132 | 0.394-02 |
| $F_1$ | 0.996 | 0.962 | 0.705 | 0.139 | 0.503-02 |

Coulombian model $\alpha(\text{v0}) = 1.241$. The corresponding results for the form factors are also shown in table 4. As expected, they get closer to those quoted as non-relativistic ones or to the exact ones. However, the form of the interaction prevents one from determining in an easy way the expression of the associated two-body currents. Keeping in mind that this extra set of results ensured a continuous transition between the results obtained with different interaction models, we will only consider in sect. 5 the simplest case, v1, for which two-body currents can also be derived without too much difficulty, while remaining close to realistic ones.

## 4 Remarks concerning form factors at low and high Q$^2$

Previous “point form” results obtained in the single-particle current approximation were found to significantly depart from the “exact” ones. We analyze on general grounds the role of further contributions to the current in correcting form factors, successively at low and high $Q^2$. In one case, they concern low-energy theorems and consistency properties, while in the other, they involve the Born amplitude.

### 4.1 Analysis of results at low Q$^2$

A detailed examination of the “point form” calculation of form factors shows that a large part of the difference with the non-relativistic calculation can be traced back to the relativistic kinematical boost effect [2]. The quantity $p_z \pm \frac{Q}{4}$, which appears in the non-relativistic Breit-frame expression of the form factor for the ground state for instance (Coulombian case)

$$I^{\text{N.R.}} \propto \int \mathrm{d}\vec{p} \left(\kappa^2 + p_x^2 + p_y^2 + \left(p_z - \frac{Q}{4}\right)^2\right)^{-2} \times \left(\kappa^2 + p_x^2 + p_y^2 + \left(p_z + \frac{Q}{4}\right)^2\right)^{-2}, \qquad (20)$$

is replaced by $(p_z \pm v\, e_p)/\sqrt{1-v^2}$ to give

$$I^{\text{P.F.}} \propto \int \mathrm{d}\vec{p} \left(\kappa^2 + p_x^2 + p_y^2 + \left(\frac{p_z - v\,e_p}{\sqrt{1-v^2}}\right)^2\right)^{-2} \times \left(\kappa^2 + p_x^2 + p_y^2 + \left(\frac{p_z + v\,e_p}{\sqrt{1-v^2}}\right)^2\right)^{-2}. \qquad (21)$$

To emphasize differences with the non-relativistic expression, we rewrite the term $(p_z \pm v\, e_p)/\sqrt{1-v^2}$ as

$$\frac{p_z \pm v\,e_p}{\sqrt{1-v^2}} = \frac{p_z}{\sqrt{1-v^2}} \pm \frac{Q}{4}\,\frac{2\sqrt{m^2+p^2}}{M}. \qquad (22)$$

This differs from the non-relativistic expression in two ways: the factor multiplying $p_z$, $1/\sqrt{1-v^2}$, and the factor multiplying $Q/4$, $(2\sqrt{m^2+p^2})/M$. They are successively analyzed in the following.

#### 4.1.1 The factor $1/\sqrt{1-v^2}$

A consequence of the relativistic boost is the appearance of the factor $1/\sqrt{1-v^2}$ multiplying $p_z$. As outlined in appendix B, a simple change of variable allows one to remove it from the integrand in eq. (21) and to factor out the quantity $\sqrt{1-v^2}$. Up to this factor and provided that the factor $(2\sqrt{m^2+p^2})/M$ can be replaced by 1 (see next section), the integral, eq. (21), then becomes identical to its non-relativistic limit, eq. (20). Notice that the result involves the very dependence of the wave function on the momentum. To evidence the ambiguous character of the above change of variables, it suffices to replace the factor at the denominator of the wave function employed in eq. (20), $\kappa^2+\vec{p}^{\,2}$, by the equivalent $\kappa^2+e_p^2-m^2$. If $e_p$ happened to combine with an interaction term, as discussed in the next subsection, to give an overall mass term, there would be no more momentum dependence and the result would be quite different. In this case, however, the replacement of the genuine momentum dependence into an energy dependence has no theoretical foundation but this may be different for other factors entering the calculation.

#### 4.1.2 The factor $(2\sqrt{m^2+p^2})/M$

The extra factor $(2\sqrt{m^2+p^2})/M$ which multiplies the quantity $Q/4$ in eq. (22) is always larger than unity, both because the numerator, $2\sqrt{m^2+p^2}$, is larger than $2\,m$ and the total mass of the system, $M$, at the denominator is smaller than the same quantity. In some sense, the momentum transfer $Q$ entering the non-relativistic calculation should be replaced by an effective one, which is larger, leading to an effective scaling of the electromagnetic properties similar to that one found in ref. [2]. The effect is especially large when the average momentum of particles composing the system under consideration is large, as is in the nucleon wave functions employed in this latter reference, or when the total mass of the system goes to zero.

Many examples indicate that one should be cautious about effects involving a factor like $(2\sqrt{m^2+p^2})/M$. Often, the kinetic energy $e_p$ combines with the potential energy $V$ to give the total energy, $M$. In such a case, results based on the above expression, or a similar one, could be affected. Illustrations include the nuclear mean-field approach, the Siegert theorem, the current in relation with symmetry arguments, the dependence of the asymptotic light-front components on the front orientation [33]. Some detail can be found in ref. [34].

Assuming that the observation made in the above examples also works in the present case, one should add in eq. (22) an interaction term $V$ so that the factor multiplying $Q/4$ now reads $(2\sqrt{m^2+p^2}+V)/M$. Taking into account that the numerator acting on a wave function is nothing but $M$, the factor may be equal to unity, as the analysis of the triangle Feynman diagram tends to show [23]. This immediately removes the scaling of electromagnetic properties mentioned at the beginning of this subsection, making the results closer to the exact and the non-relativistic ones.

The idea behind getting together those contributions is that, when a boost is made, not only the kinetic energy which enters the total mass of a system is boosted, but also the potential energy part is. As the various examples mentioned in this subsection show, one should therefore look with much caution at the present results in the "point form" approach, especially those at small $Q^2$ like the scaling of some properties with the inverse of the total mass of the system, $M$.

### 4.2 Analysis of results at high $Q^2$

At high $Q^2$ it is expected that form factors are dominated by the contribution of the full Born amplitude represented by the Feynman diagram shown in fig. 2. It is on this basis that Alabiso and Schierholz made predictions for form factors in the asymptotic domain [14]. All calculations employing wave functions obtained from some equation together with some interaction provide a contribution to the full diagram shown in this figure. Using a perturbative-type approach, this contribution can be calculated. By comparing it to the full diagram, one can determine how

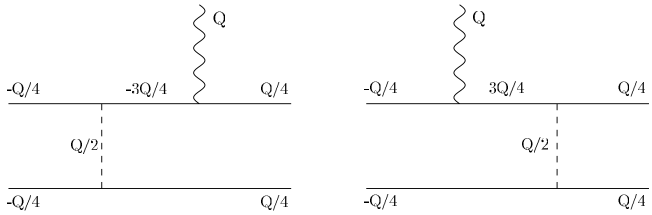


**Fig. 2.** Virtual scalar particle or photon absorption on a two-body system in Born approximation. The kinematical definitions refer to the Breit frame. They can be used for both the Feynman diagram and the non-relativistic (or instant-form) approach where the 3-momenta are conserved at all vertices, but not in the "point form" approach where a different conservation law holds.

it does in predicting the high-$Q^2$ behavior of form factors with respect to the underlying theory. What is missing may be incorporated in two-body currents. In this subsection, we analyze the contribution of each approach in the Born approximation. This follows lines developed in various papers, especially in ref. [15].

Beginning with the non-relativistic calculation for the ground state, it is found that the form factors at high $Q^2$ can be expressed as the product of the Born amplitude times the squared wave function at the origin (in configuration space):

$$\begin{aligned} F_0(q^2)|_{Q^2\to\infty} &= F_1(q^2)|_{Q^2\to\infty} \\ &= 2\,\psi^r(0)\,\psi^p\left(p=\frac{Q}{2}\right)\Big|_{Q^2\to\infty} \\ &= 2\,\mathrm{Born}\,(1\,\mathrm{diagr.})|_{Q^2\to\infty}\,\psi_r^2(0)\,. \end{aligned} \qquad (23)$$

The wave function at the origin, $\psi_r(0)$, should be determined numerically. In the Coulombian case referred to in tables 1 and 2, it is given by $\psi_r^2(0)=\kappa^3/\pi$. As for the Born amplitude in the non-relativistic case, it reads

$$\mathrm{Born}\;(1\,\mathrm{diagr.,\;N.R.})|_{Q^2\to\infty} = \frac{g^2}{\mu^2+Q^2/4}\,\frac{4\,m}{Q^2}. \qquad (24)$$

It corresponds to the product of a term involving the interaction and the propagator for the two constituent particles, both being calculated in any frame but consistently with Galilean invariance. In order to easily identify the interaction term, the mass of the exchanged boson, $\mu$, is written explicitly even though it is taken as zero in actual calculations performed later on. From the above result, it is immediately seen that the form factors scale like $1/Q^4$, factors $1/Q^2$ being contributed separately by the boson and the constituent propagators in fig. 2, in agreement with the standard counting rules for determining the high-$Q^2$ behavior of form factors. This roughly explains the behavior of form factors evidenced by the "exact" results shown in tables 1 and 2 and, of course, in the non-relativistic case.

The above result can be refined by considering the full Feynman diagrams shown in fig. 2. These ones can be split into two terms where the intermediate constituent propagates with positive and negative energies. The details may depend on the formalism or on the frame. The expressions have a form similar to eq. (23), except that scalar and charge form factors now differ and involve corrections of relativistic order:

$$F_0(q^2)\big|_{Q^2\to\infty} = 2\ (\text{B.A.}_0)|_{Q^2\to\infty}\,\frac{N}{4m}\tilde{\psi}_r^2(0),$$
$$F_1(q^2)\big|_{Q^2\to\infty} = 2\ (\text{B.A.}_1)|_{Q^2\to\infty}\,\frac{N}{E_i+E_f}\tilde{\psi}_r^2(0),$$
$$\text{with } \tilde{\psi}_r(0) = \int \frac{\mathrm{d}\vec{p}}{(2\pi)^3}\,\frac{m}{e_p}\,\phi(\vec{p}). \qquad (25)$$

The normalization constant $N$ is defined in eq. (C.2) and the factors $4m$ and $E_i+E_f$ cancel a corresponding factor in the Born amplitude such that we recover the definition of the form factors, see eq. (1). For the instant-form formalism and in the Breit frame, the Born amplitudes in the asymptotic limit read

$$\begin{aligned}\text{B.A.}_0 &= \frac{g^2}{\mu^2+Q^2/4}\,\frac{1}{2e_{3Q/4}}\\ &\quad\times\left(\frac{2m}{e_{3Q/4}-e_{Q/4}}+\frac{2m}{e_{3Q/4}+e_{Q/4}}\right)\\ &= \frac{g^2}{\mu^2+Q^2/4}\,\frac{2m}{e^2_{3Q/4}-e^2_{Q/4}},\end{aligned} \qquad (26)$$

$$\begin{aligned}\text{B.A.}_1 &= \frac{g^2}{\mu^2+Q^2/4}\,\frac{1}{2e_{3Q/4}}\\ &\quad\times\left(\frac{e_{3Q/4}+e_{Q/4}}{e_{3Q/4}-e_{Q/4}}+\frac{-e_{3Q/4}+e_{Q/4}}{e_{3Q/4}+e_{Q/4}}\right)\\ &= \frac{g^2}{\mu^2+Q^2/4}\,\frac{2e_{Q/4}}{e^2_{3Q/4}-e^2_{Q/4}}.\end{aligned} \qquad (27)$$

We omit normalization factors $m/e$ in the above equations. As mentioned previously, these ones have to be accounted for when the corresponding contributions to form factors are calculated, see eq. (25). In the non-relativistic limit, the positive energy part of the constituent propagator allows one to recover the non-relativistic result given by eq. (24). At very high $Q^2$ the total form factors are identical to the non-relativistic ones (the normalization factor $\frac{N}{4m}\tilde{\psi}_r^2(0)$, of the order of 1, put apart), but evidence a difference in the relative contributions of the different parts of the constituent propagator:

$$F_0(q^2)|_{Q^2\to\infty} = 2\,\frac{g^2}{\mu^2+Q^2/4}\,\frac{4\,m}{Q^2}\left(\frac{2}{3}+\frac{1}{3}\right)\frac{N}{4m}\tilde{\psi}_r^2(0),$$
$$F_1(q^2)|_{Q^2\to\infty} = 2\,\frac{g^2}{\mu^2+Q^2/4}\,\frac{4\,m}{Q^2}\left(\frac{4}{3}-\frac{1}{3}\right)\frac{N}{4m}\tilde{\psi}_r^2(0). \quad (28)$$

The impulse approximation calculation of asymptotic form factors in the "point form" approach could be performed along the above lines, with some modifications concerning the kinematics. The structure of the result is similar to eq. (24),

$$\text{Born } (1\,\text{diagr., P.F.})|_{Q^2\to\infty} = \frac{g^2}{\mu^2+\tilde{Q}^2/4}\,\frac{4\,m}{\tilde{Q}^2}, \qquad (29)$$

but the momentum transfer $Q^2$ is replaced by $\tilde{Q}^2 = Q^2\left[1+Q^2/(4M^2)\right]$, recovering what was obtained in ref. [8]. For the electromagnetic probe, an extra factor $(1+v^2)/(1-v^2)$ has to be added, in relation with the different coupling to the external probe. This is sufficient to explain the overall behavior of form factors shown in tables 1 and 2. Careful examination however indicates that there are other corrections than the one given in eq. (29) that give contributions with a log character.

In practice, depending on precise details in the formalism, the various contributions in eqs. (26), (27) may have a different weight but the overall result, eq. (28), should be recovered. This evidently applies to the "point form" results. In this case however, the situation is somewhat different because one has to completely rely on two-body currents to get the right asymptotic power law behavior of form factors.

## 5 Two-body currents: expressions and results

We consider here specific models for two-body currents. They are constructed via the requirement of current conservation and reproducing the Born amplitude. Methods allowing one to get these currents as well as their limitations are known [35,36]. They are adapted to our purpose in the case of the relativized model, v1. After discussing a norm correction in relation with the ratio $F_0/F_1$, which in some sense also involves two-body currents, a presentation of numerical results is made.

While the above-mentioned two-body currents have proved useful in many circumstances, it is not sure that they are sufficient in all cases. As mentioned in the introduction, recent works suggest that there may be other two-body currents to be considered when going away from the Breit-frame case or the $q^+=0$ one [26–28]. However, the covariance requirement that motivated their introduction cannot be invoked here where this property stems from the formalism itself. Moreover, there is no known constraint, like current conservation or reproducing the behavior of the Born amplitude, that can justify their consideration in the present work. Only the comparison with expected results can therefore provide some information on the relevance of related contributions within the "point form" approach.

### 5.1 Expressions of two-body currents

We already mentioned that two-body currents are required in most approaches to satisfy current conservation and could also be required to get the right Born amplitude. This second property may not be related to the first one, current conservation holding up to terms that are gauge invariant by themselves.

There are methods that allow one to derive contributions that restore current conservation but the result may not be quite satisfying, either because it misses the Born amplitude or because it is very cumbersome. Here, we favor the high-momentum-transfer region, and therefore the Born amplitude, and simplicity.

### 5.1.1 Two-body currents motivated by current conservation in the non-relativistic case

We begin with an interaction model that represents an extension of the model v1 of eq. (12) to any frame (in the instant form):

$$V_{\rm int}(\vec{p}_1,\vec{p}_2,\vec{p}_1{}',\vec{p}_2{}') = -\delta(\vec{p}_1+\vec{p}_2-\vec{p}_1{}'-\vec{p}_2{}') \times\sqrt{\frac{m}{e_{p_1}}\frac{m}{e_{p_2}}}\;\frac{g^2}{\mu^2+(\vec{p}_2-\vec{p}_2{}')^2}\;\sqrt{\frac{m}{e_{p_1'}}\frac{m}{e_{p_2'}}}. \tag{30}$$

The corresponding equation to be solved in principle generalizes eq. (10):

$$(E-e_{p_1}-e_{p_2})\,\Phi(\vec{p}_1,\vec{p}_2) = \iint \frac{{\rm d}\vec{p}_1{}'}{(2\pi)^3}\frac{{\rm d}\vec{p}_2{}'}{(2\pi)^3}V_{\rm int}(\vec{p}_1,\vec{p}_2,\vec{p}_1{}',\vec{p}_2{}')\,\Phi(\vec{p}_1{}',\vec{p}_2{}'). \tag{31}$$

Though the set of eqs. (30), (31) is not the one we will use for actual calculations, it offers the great advantage, due to its close relation to a field-theory approach, that currents take a relatively simple form, allowing one to illustrate some of the peculiarities relative to their derivation. It is noticed that the solutions for the mass $M$ only make sense in the non-relativistic limit as they in principle depend on the total momentum. A complete interaction kernel would be required to make the solutions meaningful so that to fulfill relativistic covariance. This is not however necessary for the following developments.

The single-particle current stems from the same field theory that motivates the above interactions. It is given for particle 1 by

$$J^0_{\rm I.A.} = \frac{e_{p_1}+e_{p_1'}}{2\sqrt{e_{p_1'}\,e_{p_1}}}\,\delta(\vec{q}+\vec{p}_1-\vec{p}_1{}')\,,$$
$$\vec{J}_{\rm I.A.} = \frac{\vec{p}_1+\vec{p}_1{}'}{2\sqrt{e_{p_1'}\,e_{p_1}}}\,\delta(\vec{q}+\vec{p}_1-\vec{p}_1{}'). \tag{32}$$

The above current provides a non-zero four-divergence which, using eq. (31), can be written as

$$q_\mu\!\cdot\! J^\mu_{\rm I.A.} = \delta(\vec{q}+\vec{p}_1+\vec{p}_2-\vec{p}_1{}'-\vec{p}_2{}') \times\sqrt{\frac{m}{e_{p_1'}}\frac{m}{e_{p_2'}}}\;\frac{g^2}{\mu^2+(\vec{p}_2-\vec{p}_2{}')^2}\;\sqrt{\frac{m}{e_{p_1}}\frac{m}{e_{p_2}}} \times\left(\frac{e_{p_1'}-e_{p_1'-q}}{2\,e_{p_1'-q}}+\frac{e_{p_1+q}-e_{p_1}}{2\,e_{p_1+q}}\right). \tag{33}$$

This has to be canceled by the four-divergence of a two-body current [36], which can be easily obtained in the present case:

$$\vec{J}_{\rm int}(\vec{q},\vec{p}_1,\vec{p}_2,\vec{p}_1{}',\vec{p}_2{}') = \delta(\vec{q}+\vec{p}_1+\vec{p}_2-\vec{p}_1{}'-\vec{p}_2{}') \times\sqrt{\frac{m}{e_{p_1'}}\frac{m}{e_{p_2'}}}\;\frac{g^2}{\mu^2+(\vec{p}_2-\vec{p}_2{}')^2}\;\sqrt{\frac{m}{e_{p_1}}\frac{m}{e_{p_2}}} \times\left(\frac{2\,\vec{p}_1{}'-\vec{q}}{2\,e_{p_1'-q}\,(e_{p_1'}+e_{p_1'-q})}+\frac{2\,\vec{p}_1+\vec{q}}{2\,e_{p_1+q}\,(e_{p_1}+e_{p_1+q})}\right). \tag{34}$$

Notice that one can easily recognize in this equation the structure of a pair term. By construction, it allows one to satisfy current conservation. When checking this property, it is found that the one- and two body-currents provide the following contributions (in the operatorial sense):

$$\vec{q}\!\cdot\!\vec{J}_{\rm I.A.} = (e_{p_1}\,O(1) - O(1)\,e_{p_1}),$$
$$\vec{q}\!\cdot\!\vec{J}_{\rm int} = (V\,O(1) - O(1)\,V),$$

where $O(1)$ just represents the charge operator. After adding a contribution $(e_{p_2}\,O(1) - O(1)\,e_{p_2})$, which is zero, they combine to give

$$\vec{q}\!\cdot\!(\vec{J}_{\rm I.A.}+\vec{J}_{\rm int}) = H\,O(1) - O(1)\,H, \tag{35}$$

The last equality, taken between eigen-states of the Hamiltonian, is the product of the energy transfer, $q^0$, times the charge operator. This provides another illustration of how contributions involving the kinetic energy and the potential energy separately add together to give the total energy of the system. We notice that the above two-body current does not contain any $1/q^2$ factor as some recipe enforcing current conservation,

$$J^\mu \to J^\mu - q^\mu\,J\!\cdot\! q\,/q^2, \tag{36}$$

would suppose [8]. Notice that in the small-$q$ limit, the two-body current obtained in eq. (34) has the schematic form $-\partial_{q_\mu}(\text{“}J\cdot q\text{”})$, where “$J\cdot q$” is given by the right-hand side of eq. (33). It therefore significantly differs from the term introduced in eq. (36) and, evidently, it has no singular character in the limit $q\to 0$.

### 5.1.2 Two-body currents motivated by the Born amplitude in the non-relativistic case

The contribution derived above is not sufficient to recover the Born amplitude. Starting from this requirement, another two-body current is obtained, which, underlying the theoretical model under consideration, also contributes to the time component of the current, contrary to the interaction term. The extra term to be added to eq. (34) is self-gauge invariant. To emphasize this feature, it is written in a way where this property is readily satisfied, *i.e.* by introducing the photon polarization $\epsilon^\mu$:

$$\Big(\epsilon^0\,J^0_{\Delta B} - \vec{\epsilon}\cdot\vec{J}_{\Delta B}\Big)(\vec{q},\vec{p}_1,\vec{p}_2,\vec{p}_1{}',\vec{p}_2{}') = \delta(\vec{q}+\vec{p}_1+\vec{p}_2-\vec{p}_1{}'-\vec{p}_2{}') \times\sqrt{\frac{m}{e_{p_1'}}\frac{m}{e_{p_2'}}}\;\frac{g^2}{\mu^2+(\vec{p}_2-\vec{p}_2{}')^2}\;\sqrt{\frac{m}{e_{p_1}}\frac{m}{e_{p_2}}}\;\;(\epsilon^0\,\vec{q}-\vec{\epsilon}\,q^0) \cdot\left(\frac{2\,\vec{p}_1{}'-\vec{q}}{2\,e_{p_1'-q}\,(e_{p_1'}+e_{p_1'-q})\,(e_{p_1}+e_{p_2}-e_{p_2'}+e_{p_1'-q})} - \frac{2\,\vec{p}_1+\vec{q}}{2\,e_{p_1+q}\,(e_{p_1}+e_{p_1+q})\,(e_{p_1'}+e_{p_2'}-e_{p_2}+e_{p_1+q})}\right). \tag{37}$$

While deriving these currents, off-shell effects have been neglected, which amount to corrections of the order $g^4$.

This is done consistently with neglecting higher-order contributions in the amplitude. Notice that the subscript $\Delta B$ in eq. (37) and later on does not refer to the Born amplitude itself. It represents the contribution that has to be added to the impulse approximation plus interaction terms in order to recover the Born amplitude.

While current conservation tells us nothing about two-body contributions in the case of a scalar probe, requiring that the Born amplitude provided by the Feynman diagram of fig. 2 be reproduced, imposes to consider further terms. These ones, for an interaction of particle 1 with the external probe, are given by

$$S_{\Delta B}(\vec{q},\vec{p}_1,\vec{p}_2,\vec{p}_1{}',\vec{p}_2{}') = \delta(\vec{q}+\vec{p}_1+\vec{p}_2-\vec{p}_1{}'-\vec{p}_2{}') \times \sqrt{\frac{m}{e_{p_1'}}\,\frac{m}{e_{p_2'}}}\,\frac{g^2}{\mu^2+(\vec{p}_2-\vec{p}_2{}')^2}\,\sqrt{\frac{m}{e_{p_1}}\,\frac{m}{e_{p_2}}} \times\left(\frac{2\,m}{2\,e_{p_1'-q}\,(e_{p_1}+e_{p_2}-e_{p_2'}+e_{p_1'-q})} + \frac{2\,m}{2\,e_{p_1+q}\,(e_{p_1'}+e_{p_2'}-e_{p_2}+e_{p_1+q})}\right). \tag{38}$$

It is noticed, not surprisingly, that the contributions from eqs. (37), (38) to form factors identify to the second term on the r.h.s. of eqs. (26), (27) in the same limit. This is due in part to the instant-form formalism which underlies both expressions.

### 5.1.3 Two-body currents motivated by current conservation in the "point form" approach

The first step in deriving two-body currents for calculating form factors in the "point form" is the definition of the interaction corresponding to the interaction mentioned previously. Its invariant form involves the four-velocity $\lambda^\mu$ of the system which we are interested in:

$$V_{\rm int}(p_1,p_2,p_1',p_2') = -\sqrt{\frac{m}{\lambda\cdot p_1}\,\frac{m}{\lambda\cdot p_2}}\int \mathrm{d}\eta \times\frac{g^2\,\delta^4(p_1+p_2-p_1'-p_2'-\lambda\eta)}{\mu^2-(p_2-p_2')^2+(\lambda\cdot(p_2-p_2'))^2}\,\sqrt{\frac{m}{\lambda\cdot p_1'}\,\frac{m}{\lambda\cdot p_2'}}, \tag{39}$$

with $\lambda\cdot(p_1-p_2)=\lambda\cdot(p_1'-p_2')=0$. Equation (31) then reads

$$(M-\lambda\cdot(p_1+p_2))\int \mathrm{d}\eta\delta^4(p_1+p_2-\lambda\eta)\,\Phi(p_1,p_2) = \frac{1}{(2\pi)^3} \times\iint \mathrm{d}^4p_1'\,\mathrm{d}^4p_2'\,V(p_1,p_2,p_1',p_2')\,\delta(p_1'{}^2-m^2)\,\delta(p_2'{}^2-m^2) \times(p_1'+p_2')^2\int \mathrm{d}\eta'\,\delta^4(p_1'+p_2'-\lambda\eta')\,\Phi(p_1',p_2'). \tag{40}$$

It is noticed that the extra term at the meson propagator in eq. (39), $(\lambda\cdot(p_2-p_2'))^2$, is a consequence of the kinematical character of the boost transformation in the "point form" formalism. While the mass operator corresponding to the above interaction is independent of the velocity, as it should be, the appearance of $\lambda^\mu$ is somewhat unusual from a field-theory point of view. It leads to specific off-shell effects that change the asymptotic dependence from $Q^2$ to $Q^4$ in the meson propagator appearing in the Born amplitude, see eq. (29). It partly explains the too fast drop-off of form factors calculated in the impulse approximation in the "point form" approach.

A minimal set of two-body currents can be obtained by calculating the divergence of the current accounted for in impulse approximation,

$$\frac{2\,\lambda_f^\mu\,\lambda_f\cdot p+2\,\lambda_i^\mu\,\lambda_i\cdot p-2\,p^\mu}{\sqrt{2\,\lambda_f\cdot p}\,\sqrt{2\,\lambda_i\cdot p}}\,\delta(\vec{q}+\vec{P}_i-\vec{P}_f), \tag{41}$$

and determining what is needed to recover current conservation, similarly to what was done in the non-relativistic case, see eqs. (32)-(34). Using the relation $q^\mu = M_f\,\lambda_f^\mu - M_i\,\lambda_i^\mu$, it is found that the four-divergence of the single-particle current can be expressed in terms of the interaction and is given by

$$q_\mu\cdot J^\mu_{\rm I.A.} = \sqrt{\frac{m}{\lambda_f\cdot p_{1f}}\,\frac{m}{\lambda_f\cdot p_{2f}}}\,g^2\,\delta(\vec{q}+\vec{P}_i-\vec{P}_f)\,\sqrt{\frac{m}{\lambda_i\cdot p_{1i}}\,\frac{m}{\lambda_i\cdot p_{2i}}} \times\lambda_i\cdot\lambda_f\left(\frac{\lambda_f\cdot p_{2f}}{\lambda_i\cdot p_{2f}}\,\frac{1}{H(\lambda_i)}-\frac{1}{H(\lambda_f)}\,\frac{\lambda_i\cdot p_{2i}}{\lambda_f\cdot p_{2i}}\right), \tag{42}$$

where $H(\lambda)=\mu^2-(p_{2i}-p_{2f})^2+(\lambda\cdot(p_{2i}-p_{2f}))^2$. The term of eq. (42) has to be canceled by the contribution of a two-body current, which has to be guessed for some part. A possible solution is given by

$$J^\mu_{\rm int}(q,p_{1i},p_{2i},p_{1f},p_{2f}) = \sqrt{\frac{m}{\lambda_f\cdot p_{1f}}\,\frac{m}{\lambda_f\cdot p_{2f}}}\,g^2\,\delta(\vec{q}+\vec{P}_i-\vec{P}_f)\,\sqrt{\frac{m}{\lambda_i\cdot p_{1i}}\,\frac{m}{\lambda_i\cdot p_{2i}}} \times\left[\left(\frac{(\lambda_f\cdot p_{2f})\,\lambda_f^\mu-p_{2f}^\mu}{M_i\,(\lambda_i\cdot p_{2f})\,H(\lambda_i)}+\frac{(\lambda_i\cdot p_{2i})\,\lambda_i^\mu-p_{2i}^\mu}{H(\lambda_f)\,M_f\,(\lambda_f\cdot p_{2i})}\right) +\frac{1}{H(\lambda_f)}\,X^\mu\,\frac{1}{H(\lambda_i)}\right]. \tag{43}$$

with

$$X^\mu = \Big((\lambda_i+\lambda_f)\cdot(p_{2i}-p_{2f})\Big)\left(\frac{M_f\,p_{2f}^\mu-M_i\,p_{2i}^\mu}{M_f\,M_i} -\frac{M_f\,\lambda_f^\mu+M_i\,\lambda_i^\mu}{M_f\,M_i\,(M_i+M_f)}(M_f\,\lambda_f\cdot p_{2f}-M_i\,\lambda_i\cdot p_{2i})\right). \tag{44}$$

Terms retained here resemble those obtained by using the minimal coupling principle but the output, depending on the order of the operators, is not necessarily unique, holding up to gauge-invariant terms proportional to $\lambda_i\cdot\lambda_f\,(M_f\,\lambda_f^\mu+M_i\,\lambda_i^\mu)-(M_f\,\lambda_i^\mu+M_i\,\lambda_f^\mu)$ for instance. This uncertainty especially affects the last term in eq. (44). It can be removed for a part by requiring to recover the Born amplitude (see below).

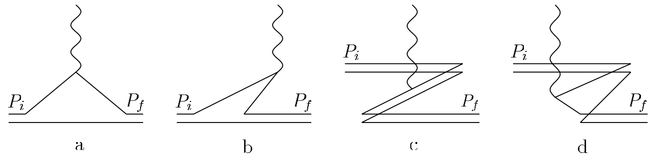


**Fig. 3.** Time-ordered triangle diagrams contributing to the absorption of a photon on a bound system.

The above current could solve some of the problems at low $Q^2$ related to current conservation. Again, we notice that it does not contain any $1/Q^2$ factor. The appearance of the total mass $M$ at the denominator in the above two-body current is not quite expected and, most probably, results from enforcing current conservation. By itself, the presence of this quantity in the current is not surprising, as it is well known that the momentum in the "point form" approach, and therefore the current, depends on the interaction. However, it can be checked that the above current does not help in solving the vanishing of the elastic scalar form factor in the limit $M \to 0$, which also shows up at small momentum transfers.

### 5.1.4 Two-body currents motivated by the ratio $F_0/F_1$ in the limit $M \to 0$

While considering the problem of the ratio $F_0/F_1$ in the limit $M \to 0$, it is useful to examine the simplest triangle Feynman diagram describing the interaction of a two-body system with an external probe. This one splits into six different time-ordered diagrams, partly shown in fig. 3. The first of them (a) represents the standard contribution that possibly involves the wave function of the system. The second one (b) is a typical Z-type diagram. The third and fourth ones (c and d) are less known. They involve two particles with negative energy or, equivalently, going backward in time. Looking now at the problem raised by the ratio $F_0/F_1$, we found that the required two-body currents also contribute to the norm, defined by the charge associated with the conserved current, $F_1(q^2 = 0) = 1$. This differs from standard two-body currents, in the two-nucleon system for instance, or in eq. (34), which do not contribute to the charge (in the usual approach where the interaction does not depend on the energy). The extra currents involve the double Z-diagram shown in fig. 3c. They are not especially suppressed for the scalar coupling model considered here and they contribute destructively to the norm so that to cancel the normal contribution (fig. 3a) in the limit $M \to 0$. Such a result can be checked by calculating the contribution of the triangle diagram of the figure in the case where the momentum dependence of the bound-state vertex function is ignored (see also ref. [36]):

$$\int \mathrm{d}^4p \, \frac{1}{m^2-(P_i-p)^2-i\epsilon} \, \frac{1}{m^2-(P_f-p)^2-i\epsilon} \times \frac{P_i^0+P_f^0-2\,p^0}{m^2-p^2-i\epsilon} = 2i\pi \int \mathrm{d}^3p \, \frac{1}{(2\,e_p)^3} \left( \frac{2\,e_p}{(2\,e_p-M)^2} - \frac{2\,e_p}{(2\,e_p+M)^2} \right), \quad (45)$$

where $P_i^0$ and $P_f^0$ are expressed in the c.m. system, $P_i^0 = P_f^0 = M$. For the scalar case, one gets

$$\int \mathrm{d}^4p \, \frac{1}{m^2-(P_i-p)^2-i\epsilon} \, \frac{1}{m^2-(P_f-p)^2-i\epsilon} \times \frac{2\,m}{m^2-p^2-i\epsilon} = 2i\pi \int \mathrm{d}^3p \, \frac{1}{(2\,e_p)^3} \left( \frac{2\,m}{(2\,e_p-M)^2} + \frac{4\,m}{2\,e_p\,(2\,e_p-M)} + \frac{4\,m}{2\,e_p\,(2\,e_p+M)} + \frac{2\,m}{(2\,e_p+M)^2} \right). \quad (46)$$

In eqs. (45), (46), the first term on the r.h.s. represents the standard non-relativistic contribution. The extra terms, which should also occur in the instant form of relativistic quantum mechanics, greatly complicate the calculation of form factors. They cannot be neglected however and, in fact, their introduction seems to provide more consistency in the developments. The matrix element of the current in the "point form" approach, eq. (15), appears to be proportional to the factor $M$ because this one has been introduced as an overall factor. In the Bethe-Salpeter approach, eq. (3), this factor appears dynamically. The fact that the four-vector current matrix element should be proportional to $P_i^\mu + P_f^\mu$ automatically ensures that it is proportional to $M$ at $\vec{P} = 0$ (without requiring the introduction of this factor by hand). This is just a consequence of the extra current discussed above.

At the same time as the front factor $M$ is removed from the r.h.s. of eq. (15), it disappears from the expression of the scalar form factor, eq. (14) and eq. (17). This form factor does not vanish anymore in the limit $M \to 0$. The ratio $F_0(0)/F_1(0)$ stemming from eqs. (45), (46) is 1.5 in the limit $M \to 0$, while the Wick-Cutkosky result is 1.25 [3]. Finally, to recover the full Born amplitude, one can use the standard definition of the current without renormalizing its expression, somewhat arbitrarily, by a factor $1/M$ in order to compensate the front factor $M$ in eqs. (14), (17).

The discussion of the above contributions would require a full paper by itself and, as far as we can see, they do not help in solving the current conservation problem considered in this subsection. In practice, we will account for them by multiplying the single-particle current operator by a constant factor suggested by the expression of eq. (45):

$$F = 1 - \left( \frac{M - 2\,\bar{e}}{M + 2\,\bar{e}} \right)^2 = \frac{8\,M\,\bar{e}}{(M+2\,\bar{e})^2}, \quad (47)$$

where $\bar{e}$ represents an average value of $\sqrt{m^2+p^2}$. The two-body nature of the correction is not explicit, but corresponding to an off-shell effect, it can be made transparent by expressing the factor $M - 2e$ in terms of the

[3] Notice that the light-front approach seems to do correctly with respect to this problem. The ratio $F_0(0)/F_1(0)$ is finite in the limit $M \to 0$ and its value, 7/6, is close to the expected one. Furthermore, a contribution like the double Z-diagram of fig. 3c vanishes in this approach for the model considered here.

interaction using eq. (31). The correction is at least of the second order in this interaction and while its effect is small for weakly bound systems, it it certainly considerable when the total mass goes to zero. Interestingly, the above example gives a further illustration of how a term proportional to the kinetic energy, $e_p$ (first term on the r.h.s. of eq. (45)) turns into a term proportional to the total mass by incorporating interaction effects.

The above approximation would certainly be questionable for an exact calculation but, being interested in whether some of the striking features evidenced by the impulse “point form” results can be repaired for some part by adding two-body currents, we believe it should not affect the developments presented below. On the other hand, it allows one to continue to work with a conserved current, including the single- and the two-body parts given by eqs. (43), (44).

#### 5.1.5 Two-body currents motivated by the Born amplitude in the “point form” approach

When considering the further requirement of reproducing the Born amplitude, an extra contribution arises. This one is obtained by subtracting from this amplitude the contribution accounted for in the impulse approximation calculation, eqs. (14), (15), and that one accounting for current conservation, eqs. (43), (44) (see also appendix D). As current conservation holds for the Born amplitude, it also holds for the above difference in the same limit, *i.e.* at the order $g^2$. Neglected contributions are of the order $g^4$, which in any case are discarded when limiting ourselves to the Born amplitude. A few details are given in appendix D; here we give an expression where the two-body contribution is written in a way where current conservation is manifest, analogously to eq. (37):

$$\begin{aligned}
&\epsilon_\mu \cdot J^\mu_{\Delta B}(q, p_{1i}, p_{2i}, p_{1f}, p_{2f}) = \\
&\sqrt{\frac{m}{\lambda_f \cdot p_{1f}} \frac{m}{\lambda_f \cdot p_{2f}}} \frac{g^2}{4} \delta(\vec{q} + \vec{P}_i - \vec{P}_f) \sqrt{\frac{m}{\lambda_i \cdot p_{1i}} \frac{m}{\lambda_i \cdot p_{2i}}} \\
&\times \frac{\mu^2 - (p_{2i} - p_{2f})^2 + \lambda_i \cdot (p_{2i} - p_{2f}) \, \lambda_f \cdot (p_{2f} - p_{2i})}{\lambda_f \cdot p_{2f} \, \lambda_i \cdot p_{2i} \, H(0) \, H(\lambda_f) H(\lambda_i)} \\
&\times \frac{2 \, \bar{e}}{M} \epsilon_\mu \cdot \Big( Y^\mu (p_{2i} - p_{2f}) \cdot q - (p_{2i} - p_{2f})^\mu \, Y \cdot q \Big), \qquad (48)
\end{aligned}$$

with

$$Y^\mu = \lambda^\mu_f \, \lambda_f \cdot p_{2f} + \lambda^\mu_i \, \lambda_i \cdot p_{2i} - \frac{1}{2}(p^\mu_{2f} + p^\mu_{2i}). \qquad (49)$$

In deriving this expression, we assume the relationship

$$q^\mu = \lambda^\mu_f \, M_f - \lambda^\mu_i \, M_i \simeq \frac{M}{\bar{e}} \, (\lambda^\mu_f \, \lambda_f \cdot p_{2f} - \lambda^\mu_i \, \lambda_i \cdot p_{2i}), \quad (50)$$

where the replacement of $\lambda \cdot p$ by an average value $\bar{e}$ is in accordance with neglecting contributions of order higher in $g^2$.

For the scalar probe, the extra contribution required to reproduce the Born amplitude is given by

$$\begin{aligned}
&S_{\Delta B}(q, p_{1i}, p_{2i}, p_{1f}, p_{2f}) = \\
&\sqrt{\frac{m}{\lambda_f \cdot p_{1f}} \frac{m}{\lambda_f \cdot p_{2f}}} \frac{g^2}{2} \, \delta(\vec{q} + \vec{P}_i - \vec{P}_f) \sqrt{\frac{m}{\lambda_i \cdot p_{1i}} \frac{m}{\lambda_i \cdot p_{2i}}} \\
&\times \Bigg[ \frac{1}{H(0)} \left( \frac{m}{\lambda_i \cdot p_{2i} \, \lambda_i \cdot p_{2f}} + \frac{m}{\lambda_f \cdot p_{2f} \, \lambda_f \cdot p_{2i}} \right) \\
&- \left( \frac{m \, \lambda_i \cdot (p_{2i} - p_{2f})}{\lambda_i \cdot p_{2f} \, H(0) \, H(\lambda_i)} + \frac{m \, \lambda_f \cdot (p_{2f} - p_{2i})}{\lambda_f \cdot p_{2i} \, H(0) \, H(\lambda_f)} \right) \Bigg]. \quad (51)
\end{aligned}$$

Notice that eqs. (48), (51) correspond to contributions to the Born amplitude that are not generated in another way and that there is therefore no double counting. Examination of the two-body currents, eqs. (43), (44) and (48), (51), which involve a neutral-boson exchange, shows that they are much more sophisticated than standard ones in the same case. They exhibit unusual features, like the appearance of the boson propagator twice, which generally characterizes the contribution of a charged boson interacting with an external field.

### 5.2 Results involving two-body currents

In this subsection, we present expressions for form factors incorporating contributions from two-body currents. These ones are motivated by fulfilling current conservation and reproducing the Born amplitude. They are followed by two sets of results, obtained with a Galilean boost and the “point form” one. In both cases, the wave function from the model v1 is used.

#### 5.2.1 Expressions of the calculated form factors

The two body-currents employed with the Galilean boost are inspired from eqs. (34), (37) for the parts required to fulfill current conservation and reproduce the Born amplitude, respectively. However, since these currents are appropriate to an instant-form formalism, we use an alternative expression of these currents that we could derive assuming a Galilean-invariant extension of the interaction model v1. These ones miss the relation to a well-defined field-theory–motivated current and evidence features that are sometimes unusual with this respect, although a relation to pair-type currents can be recovered in some limit. As they are not of fundamental importance and perhaps too specialized, we prefer to give their expressions together with the contributions to form factors in appendix C.

For the “point form” approach, results are obtained from eqs. (43), (44), (47), (48), (51). The full expressions of the different form factors, including two-body currents, are given below, while some intermediate steps are given in appendix D. Using the relation implied by the $\delta^4$ functions, the momenta relative to particle number 1 can always be written in terms of the momenta relative to particle number 2 (the spectator particle) and the four-velocity vector $\lambda^\mu$. In absence of ambiguity, the momentum $\vec{p}$ in

**Table 5.** Elastic and inelastic form factors $F_0(q^2)$, $F_1(q^2)$ and $F_2(q^2)$ in a Galilean approach as given in appendix C: Effect of two-body currents motivated by current conservation ("int" line) and the Born amplitude ("int" + $\Delta B$ line). Calculations are performed with the interaction model v1 and correspond to a coupling $\alpha = 3$ for the Wick-Cutkosky model.

| $Q^2/m^2$ | 0.01 | 0.1 | 1.0 | 10.0 | 100.0 |
|---|---|---|---|---|---|
| $\alpha = 3$, elastic, int | | | | | |
| $F_0$ | 0.735 | 0.701 | 0.454 | 0.365-01 | 0.198-03 |
| $F_1$ | 0.995 | 0.953 | 0.640 | 0.665-01 | 0.774-03 |
| $\alpha = 3$, elastic, int + $\Delta B$ | | | | | |
| $F_0$ | 0.959 | 0.921 | 0.628 | 0.635-01 | 0.643-03 |
| $F_1$ | 0.996 | 0.953 | 0.641 | 0.680-01 | 0.898-03 |
| $\alpha = 3$, inelastic, int | | | | | |
| $F_0$ | −0.325-01 | 0.202-02 | 0.132-00 | 0.189-01 | 0.91-04 |
| $F_1$ | 0.469-02 | 0.430-01 | 0.186-00 | 0.318-01 | 0.33-03 |
| $F_2$ | 0.483 | 0.442 | 0.191-00 | 0.327-02 | 0.34-05 |
| (two-body part of $F_2$) | | | | | |
| | (24%) | (25%) | (28%) | (48%) | (116%) |
| $\alpha = 3$, inelastic, int + $\Delta B$ | | | | | |
| $F_0$ | 0.086-01 | 0.509-01 | 0.209-00 | 0.361-01 | 0.37-03 |
| $F_1$ | 0.468-02 | 0.429-01 | 0.185-00 | 0.325-01 | 0.41-03 |
| $F_2$ | 0.481 | 0.442 | 0.191-00 | 0.335-02 | 0.42-05 |

the expressions given below will refer to this spectator particle. When specialized to the frame $\vec{v} = \vec{v}_f = -\vec{v}_i$, the form factors $F_0(q^2)$, $\tilde{F}_1(q^2)$ and $\tilde{F}_2(q^2)$ successively read

$$F_0(q^2) = \frac{\sqrt{N_f\,N_i}}{4\,m}\left(\int \frac{\mathrm{d}\vec{p}}{(2\pi)^3}\,\phi_f(\vec{p}_{tf})\,\frac{m}{e_p}\,\phi_i(\vec{p}_{ti}) + \int \frac{\mathrm{d}\vec{p}_f\,\mathrm{d}\vec{p}_i}{(2\pi)^6}\,\phi_f(\vec{p}_{tf})\,\phi_i(\vec{p}_{ti})\,\frac{m^2}{e_f\,e_i}\,(K_{\Delta B})_0\right),$$

$$\tilde{F}_1(q^2) = \frac{\sqrt{N_f\,N_i}}{2\,\bar{M}}\sqrt{1-v^2} \times\left(\bar{F}\,\frac{1+v^2}{1-v^2}\int \frac{\mathrm{d}\vec{p}}{(2\pi)^3}\,\phi_f(\vec{p}_{tf})\,\phi_i(\vec{p}_{ti}) + \bar{F}\int \frac{\mathrm{d}\vec{p}_f\,\mathrm{d}\vec{p}_i}{(2\pi)^6}\,\phi_f(\vec{p}_{tf})\,\phi_i(\vec{p}_{ti})\,\frac{m^2}{e_f\,e_i}\,(K_{\mathrm{int}})_1 + \int \frac{\mathrm{d}\vec{p}_f\,\mathrm{d}\vec{p}_i}{(2\pi)^6}\,\phi_f(\vec{p}_{tf})\,\phi_i(\vec{p}_{ti})\,\frac{m^2}{e_f\,e_i}\,(K_{\Delta B})_1\right),$$

$$\tilde{F}_2(q^2)\,\vec{v} = -\frac{\sqrt{N_f\,N_i}}{2\,\bar{M}}\sqrt{1-v^2} \times\left(\bar{F}\,\frac{1+v^2}{1-v^2}\int \frac{\mathrm{d}\vec{p}}{(2\pi)^3}\,\phi_f(\vec{p}_{tf})\,\frac{\vec{p}}{e_p}\,\phi_i(\vec{p}_{ti}) + \bar{F}\int \frac{\mathrm{d}\vec{p}_f\,\mathrm{d}\vec{p}_i}{(2\pi)^6}\,\phi_f(\vec{p}_{tf})\,\phi_i(\vec{p}_{ti})\,\frac{m^2}{e_f\,e_i}\,(\vec{K}_{\mathrm{int}})_2 + \int \frac{\mathrm{d}\vec{p}_f\,\mathrm{d}\vec{p}_i}{(2\pi)^6}\,\phi_f(\vec{p}_{tf})\,\phi_i(\vec{p}_{ti})\,\frac{m^2}{e_f\,e_i}\,(\vec{K}_{\Delta B})_2\right), \quad (52)$$

where $\bar{M}$, $\bar{F}$ and $N$ are defined in appendix C. The expressions of the $K$ quantities, which account for two-body currents, are given by

$$(K_{\mathrm{int}})_1 = g^2\left[\left(\frac{(\lambda_f\cdot p_f)\,\lambda_f^0 - p_f^0}{M_i\,(\lambda_i\cdot p_f)\,H(\lambda_i)} + \frac{(\lambda_i\cdot p_i)\,\lambda_i^0 - p_i^0}{M_f\,H(\lambda_f)\,(\lambda_f\cdot p_i)}\right) + \frac{1}{H(\lambda_f)}\,X^0\,\frac{1}{H(\lambda_i)}\right],$$

$$(\vec{K}_{\mathrm{int}})_2 = g^2\left[\left(\frac{(\lambda_f\cdot p_f)\,\vec{\lambda}_f - \vec{p}_f}{M_i\,(\lambda_i\cdot p_f)\,H(\lambda_i)} + \frac{(\lambda_i\cdot p_i)\,\vec{\lambda}_i - \vec{p}_i}{M_f\,H(\lambda_f)\,(\lambda_f\cdot p_i)}\right) + \frac{1}{H(\lambda_f)}\,\vec{X}\,\frac{1}{H(\lambda_i)}\right],$$

$$(K_{\Delta B})_0 = \frac{g^2}{2}\left[\frac{1}{H(0)}\left(\frac{m}{\lambda_i\cdot p_i\,\lambda_i\cdot p_f} + \frac{m}{\lambda_f\cdot p_f\,\lambda_f\cdot p_i}\right) - \left(\frac{m\,\lambda_i\cdot(p_i-p_f)}{\lambda_i\cdot p_f\,H(0)\,H(\lambda_i)} + \frac{m\,\lambda_f\cdot(p_f-p_i)}{\lambda_f\cdot p_i\,H(0)\,H(\lambda_f)}\right)\right],$$

$$(K_{\Delta B})_1 = \frac{g^2}{4}\,\frac{\mu^2-(p_i-p_f)^2+\lambda_i\cdot(p_i-p_f)\,\lambda_f\cdot(p_f-p_i)}{\lambda_f\cdot p_f\,\lambda_i\cdot p_i\,H(0)\,H(\lambda_f)\,H(\lambda_i)} \times\frac{2\,\bar{e}}{\bar{M}}\left(Y^0\,(p_i-p_f)\cdot q - (p_i-p_f)^0\,Y\cdot q\right),$$

$$(\vec{K}_{\Delta B})_2 = \frac{g^2}{4}\,\frac{\mu^2-(p_i-p_f)^2+\lambda_i\cdot(p_i-p_f)\,\lambda_f\cdot(p_f-p_i)}{\lambda_f\cdot p_f\,\lambda_i\cdot p_i\,H(0)\,H(\lambda_f)\,H(\lambda_i)} \times\frac{2\,\bar{e}}{\bar{M}}\left(\vec{Y}\,(p_i-p_f)\cdot q - (\vec{p}_i-\vec{p}_f)\,Y\cdot q\right). \quad (53)$$

As can be observed, the expression of the currents required to ensure current conservation ($K_{\mathrm{int}}, \vec{K}_{\mathrm{int}}$) does not contain any $1/q^2$ factor as the recipe given by eq. (36) would imply.

#### 5.2.2 Results with two-body currents for the form factors in a Galilean approach

The non-relativistic–type calculations of table 5 are especially useful to make the transition from the "exact" results presented in sect. 3 to the "point form" ones presented in table 6, allowing one to distinguish effects specific of this last approach from those due to the restoration of current conservation, to the Born-amplitude constraint or to the dynamics.

At low momentum transfers, the suppression of the elastic form factor $F_0$ with respect to $F_1$ ("int" case) is reminiscent for some part of the one mentioned in sect. 3 for the "point form" case as a result of the normalization definition. It is largely canceled by a pair term contribution to the scalar form factor included in the Born-constrained current. The same contribution also changes the scalar inelastic form factor $F_0$ from a negative to a positive value. In both cases, the results (0.959 and 0.009 at $Q^2/m^2 = 0.01$) become closer to the "exact" ones (1.123 and 0.054). At high momentum transfers, the elastic and inelastic form factors, $F_0$, in the "int" case decrease more quickly than the corresponding vector form factors, $F_1$. This is due to an extra $(1/Q)$-dependence in the former. This discrepancy tends to disappear when the Born-constrained current is considered. The results so obtained are qualitatively in better agreement with the "exact" ones. However the magnitude is smaller by a factor 5 or so. The comparison with the Coulombian-type results suggests that the discrepancy is to be ascribed to a large part to the difference in the wave functions at the origin that, as already mentioned, determines the overall coefficient multiplying the asymptotic power law behavior of form factors. This points to the relative simplicity of the interaction model, v1, which does not do quite well as to the description of the spectrum of the Wick-Cutkosky model, while the Coulombian one does. We do not expect this discrepancy to be removed when looking at the "point form" results.

The last comment we want to make concerns the current conservation that is better seen by looking at the form factor, $F_2$. Contrary to the Born-constrained current, the two-body current motivated by current conservation has a big influence in the "int" case. From 25% of $F_2$ at low momentum transfers, its contribution can raise up to 50% for higher momentum transfers. Larger contributions are expected when the average momentum of the constituent particles increases ($\bar{p}^2/m^2 = 0.2$ in the present case).

#### 5.2.3 Results with two-body currents for the form factors in the "point form" approach

The least that one can say about the contributions of two-body currents in the "point form" approach is that they strongly depend on the coupling, the mass of the system, the elastic or inelastic, scalar or vector character of the transition, and the presence or absence of a node in the impulse approximation. We successively consider the effect of two-body currents motivated by current conservation and the Born-amplitude behavior, the corresponding results being given in table 6 [4]. These results may be compared to the "exact" (BS) ones given in tables 1-3 or to those given in table 5. These last ones may represent better benchmark results, taking into account that the v1 model, due to its simplicity, is not doing too well in reproducing the "exact" (BS) spectrum, as already mentioned.

**Table 6.** Elastic and inelastic form factors $F_0(q^2)$, $F_1(q^2)$ and $F_2(q^2)$, calculated in the "point form" approach and including two-body currents: results of eqs. (52), (53) are calculated with the interaction model, v1, and are given for different couplings of the Wick-Cutkosky model, $\alpha = 3$ ($\alpha$(v1) $= 1.775$, $E = 0.432\,m$) and $\alpha \simeq 2\,\pi$ ($\alpha$(v1) $= 5.287$, $E = 1.90\,m$). Different approximations about the two-body currents are considered, successively: impulse approximation (I.A.), with inclusion of interaction currents (int) and with inclusion of both interaction and Born-motivated currents (int $+ \Delta B$). Present results may be compared to the "exact" (B.S.) ones given in tables 1-3 or, better, to those given in table 5 (see text).

| $Q^2/m^2$ | 0.01 | 0.1 | 1.0 | 10.0 | 100.0 |
|---|---|---|---|---|---|
| $\alpha = 3$, elastic, I.A. | | | | | |
| $F_0$ | 0.732 | 0.671 | 0.312 | 0.061-01 | 0.29-05 |
| $F_1$ | 0.992 | 0.924 | 0.493 | 0.205-01 | 0.46-04 |
| $\alpha = 3$, elastic, int | | | | | |
| $F_0$ | 0.732 | 0.671 | 0.312 | 0.061-01 | 0.29-05 |
| $F_1$ | 0.992 | 0.930 | 0.521 | 0.295-01 | 0.11-03 |
| $\alpha = 3$, elastic, int $+ \Delta B$ | | | | | |
| $F_0$ | 1.142 | 1.065 | 0.573 | 0.268-01 | 0.20-03 |
| $F_1$ | 0.994 | 0.944 | 0.595 | 0.491-01 | 0.43-03 |
| $\alpha = 3$, inelastic, I.A. | | | | | |
| $F_0$ | 0.011 | 0.043 | 0.116 | 0.489-02 | 0.26-05 |
| $F_1$ | 0.019 | 0.059 | 0.167 | 0.136-01 | 0.33-04 |
| $F_2$ | 0.383 | 0.335 | 0.102 | −0.373-03 | −0.34-05 |
| $\alpha = 3$, inelastic, int | | | | | |
| $F_0$ | 0.011 | 0.043 | 0.116 | 0.489-02 | 0.26-05 |
| $F_1$ | 0.006 | 0.047 | 0.168 | 0.181-01 | 0.73-04 |
| $F_2$ | 0.535 | 0.474 | 0.171 | 0.186-02 | 0.79-06 |
| $\alpha = 3$, inelastic, int $+ \Delta B$ | | | | | |
| $F_0$ | 0.059 | 0.097 | 0.187 | 0.167-01 | 0.12-03 |
| $F_1$ | 0.005 | 0.043 | 0.170 | 0.274-01 | 0.24-03 |
| $F_2$ | 0.469 | 0.432 | 0.173 | 0.282-02 | 0.25-05 |
| $\alpha \simeq 2\,\pi$, elastic, I.A. | | | | | |
| $F_0$ | 0.096 | 0.012-01 | 0.50-06 | 0.86-10 | 0.12-13 |
| $F_1$ | 0.486 | 0.167-01 | 0.34-04 | 0.37-07 | 0.37-10 |
| $\alpha \simeq 2\,\pi$, elastic, int | | | | | |
| $F_0$ | 0.096 | 0.001 | 0.50-06 | 0.86-10 | 0.12-13 |
| $F_1$ | 3.831 | 0.460 | 0.20-02 | 0.23-05 | 0.24-08 |
| $\alpha \simeq 2\,\pi$, elastic, int $+ \Delta B$ | | | | | |
| $F_0$ | 0.359 | 0.012 | 0.92-04 | 0.87-06 | 0.87-08 |
| $F_1$ | 5.452 | 0.699 | 0.49-02 | 0.32-04 | 0.30-06 |

[4] Involving a 4-dimensional integration, the two-body part of the form factors involves some uncertainty and the accuracy of the results may be smaller than what the number of digits suggests. Differences may be significant however.

Beginning with the elastic form factor corresponding to $\alpha = 3$, we found that the effect of two-body currents motivated by current conservation is quite small at low $Q^2$ but, being constrained to go to 0 at zero momentum transfer with $Q^2$, this result is not very significant. At higher $Q^2$, the effect is much larger (40% of the impulse approximation at $Q^2 = 10\,m^2$, more than 100% at $Q^2 = 100\,m^2$). In comparison to the results given in table 5 in a similar case, we nevertheless notice that the drop-off of the form factor is faster, scaling like $Q^{-6}$ rather than $Q^{-4}$ at high $Q^2$. The inelastic form factor is more instructive on the effects at low $Q^2$ where $F_2(Q^2)$ is enhanced by about 30% while $F_1(Q^2)$ gets decreased by 100% or so. The large effects in this case are strongly related to current conservation which implies that $F_1(Q^2)$ scales like $Q^2$ at small momentum transfer. At higher momentum transfer, 100% effects, constructive or destructive, are also found. The last case now concerns the $F_2(Q^2)$ form factor which is seen to scale like $Q^{-8}$ while $F_1(Q^2)$ scales like $Q^{-6}$. In comparison to the Galilean calculation presented in table 5, the relative size of the corrections is qualitatively the same. As for the elastic case, the form factor $F_1(Q^2)$ drops too fast ($Q^{-6}$ instead of $Q^{-4}$). The results for the strongly interacting case ($M = 0.1$) provide a useful and complementary information. The corrections make the form factor overshoot the value $F_1(Q^2 = 0) = 1$. Essentially, depending on the quantity $v^2$, the corrections scale like $M^{-2}$. They are therefore enhanced in the small mass limit, in the same way that the charge radius is in the present "point form" approach. The correction drops off quickly at higher $Q^2$ but this shows the limitations that underlie the derivation of two-body currents to which too much is asked in the present case. In our effort to construct some systematics, we could have considered the inelastic form factor for $\alpha \simeq 2\,\pi$. Compared to $\alpha = 3$, we expect the effect of two-body currents to be enhanced. However, the striking features evidenced by the elastic form factors in this limiting case, corresponding to $M \to 0$, will be largely absent due to the fact that the excited state, necessarily, has a non-zero mass. In this respect, the inelastic case, $\alpha \simeq 2\,\pi$, is much less instructive than the elastic one.

All the above results drop off too fast in comparison to the expected power law behavior. This one therefore relies on the two-body currents added to reproduce the Born amplitude. At high $Q^2$, the results so obtained are however below the "exact" ones given in tables 1-3. Part of the effect is certainly due to the choice of the interaction v1. The comparison with results given in table 5, obtained with the same interaction model, should be more adequate but it still shows some discrepancy. The results for the strongly interacting case are again useful here. The suppression of the form factor at high $Q^2$ by many orders of magnitude points to a dependence of the asymptotic form factor on a factor $M^4$, which simply stems from the dependence of the form factor on the velocity $v$, which involves the factor $Q/M$. The reduction factor $(M/(2\bar{e}))^4$ largely explains the difference in the results given in tables 5 and 6 for $\alpha = 3$. As reminded in the beginning of sect. 4, one expects such factors to be canceled by interaction effects which, thus, do not appear to be accounted for by the two-body currents we looked at. The currents discussed here also contribute at low $Q^2$. The most significant effect concerns the scalar form factor. The contributions, respectively 0.4 and 0.05 for the elastic and inelastic form factors for $\alpha = 3$ compare with those obtained in the Galilean calculation, 0.22 and 0.04 (see table 5).

While looking at two-body currents motivated by current conservation, an interesting question was whether they could allow one to get the low-$Q^2$ charge form factor right, in relation with the charge radius. Results for the strong-interaction case leave this possibility open but those for the smaller coupling, $\alpha = 3$, rather point to a different answer. In this case, which is under better control (not much extra currents needed), most of the correction at low $Q^2$ is produced by the term introduced to ensure the right power law behavior.

The fact that the present two-body currents do not allow one to get close to the "exact" results raises the question of their derivation. As the standard front- and instant-form calculations give reasonable results, one could apply the unitary transformation that links different forms. Assuming first that this is tractable, there is still a principle difficulty. The instant- and front-form calculations are usually done in a particular frame (Breit frame and $q^+ = 0$, respectively), and, strictly speaking, the form factors so obtained are not Lorentz invariant. Unless some trick is employed, there is no way to relate these form factors to the explicitly Lorentz-invariant form factors obtained in "point form". Moreover, such a unitary transformation would not ensure that the two constraints we considered (current conservation and reproduction of the behavior of the Born amplitude) would be fulfilled. These ones represent in our opinion more important benchmarks, with a clear theoretical insight. Interestingly, two-body currents constructed along the above lines seem to do a good job in the case of instant-form calculations of form factors in the Breit frame (some indication can be obtained from examining results in table 5). This suggests that the above requirements are on the good track but, evidently, are not sufficient here.

## 6 Conclusion

In this work, we further investigated the calculation of form factors in the "point form" approach for a two-body system. With respect to a previous work [17], we also considered a scalar form factor, which gives another information that can be compared to a more elaborate calculation. This one is provided by the Wick-Cutkosky model where form factors can be calculated exactly as far as one neglects mass and vertex renormalization as usually done when dealing with a two-body system. Possible corrections, that could be accounted for by introducing form factors for the constituents, cancel out in this case but should be incorporated when a comparison to experiment is done. The consideration of the scalar form factor confirms conclusions reached previously. Taking into account that the non-relativistic calculation does rather well (which by itself deserves some explanation), the implementation of the "point form" approach in the impulse approximation, as used in recent works, does badly both at low and large $Q^2$. At low $Q^2$, results are not protected by the conservation of some charge as for the vector case. At large $Q^2$ it decreases more quickly, $1/Q^8$ instead of $1/Q^4$ (up to log factors). We also checked that, qualitatively, the results for both the scalar and vector form factors were unchanged by using another interaction in the mass operator. The limit $M \to 0$ reveals that the charge and scalar radii scale like $1/M$ in the "point form" approach, while the scalar form factor tends to 0 when $Q^2 \to 0$. None of these features is supported by the "exact" calculation.

Within the implementation of the "point form" approach used here, the only way to explain the above discrepancies relies on large contributions from two-body currents. These ones have been derived to ensure both current conservation and to reproduce the power law behavior of the Born amplitude. This has been done consistently with the choice for the single-particle current (the simplest one) and for an interaction model which allows for a derivation in closed form (the interaction only appears at first order). We will not comment on their performance with respect to the above-requested properties since they have been derived in such a way that they should be fulfilled. It is more interesting to investigate the qualitative and quantitative features that these two-body currents evidence.

What characterizes the two-body currents considered here is that they have not much to do with standard ones. Even though they involve the exchange of a neutral boson, and despite the fact that the interaction model has been chosen to make them as simple as possible, they are considerably more complicated than similar currents in a non-relativistic approach. Obtained somewhat by "brute force", these currents cannot be related to time-ordered diagrams. The reason for this is simple. The initial and final states being described on different hyper-planes, corresponding therefore to different invariant times, there is no way to define time-ordered diagrams unambiguously. The situation is different for the two-body currents ensuring the right $F_0/F_1$ ratio. Treated very approximately, these currents should also be a part of other relativistic quantum mechanics approaches.

To ensure current conservation, a recipe is sometimes used in the literature [8] that involves a pole at $Q^2 = 0$. This can be acceptable in the case where the spectrum in the $t$-channel exhibits a zero-mass particle, as would be the case for the axial current in hadronic physics, where this particle could be the pion in the chiral symmetry limit. The two-body currents derived here do not evidence such a pole, as expected. Actually, using the above recipe, one could construct a single- and a two-body current separately conserved. When the equation of motion is used however, it turns out that the terms with a pole at $Q^2 = 0$ cancel each other. This is an important constraint on the derivation of the two-body currents.

Quantitatively, for a rather moderately bound system ($\bar{p}^2/m^2 = 0.2$), we found that two-body currents motivated by current conservation produce contributions to elastic charge form factors ranging from 0% at low $Q^2$ up to 100% at $Q^2 = 100\,m^2$. The situation is slightly different for an inelastic transition where there is no constraint like charge conservation which imposes corrections to vanish at $Q^2 = 0$. There are other constraints and corrections that can reach 100% with a destructive character so that to recover relations such as $F_1(Q^2) \to 0$ when $Q^2 \to 0$ or $F_2(Q^2)/F_1(Q^2) \to Q^{-2}$ at high $Q^2$. Otherwise corrections vary from 25% at low $Q^2$ for $F_2(Q^2)$ up to 100% at $Q^2 = 100\,m^2$ for $F_1(Q^2)$. Corrections due to two-body currents motivated by the Born amplitude are important too and are the dominant ones beyond $Q^2 = 10\,m^2$. If one puts apart aspects specific to the "point form" approach (like the faster drop-off), large similarities with a Galilean-type calculation are observed. Evidently, larger effects may be obtained in a stronger bound system with a larger value of $\bar{p}^2/m^2$, as evidenced by some results for the case $M = 0.1\,m$.

In comparison with an "exact" calculation, present "point form" results with incorporation of contributions due to two-body currents still show striking discrepancies. The increase of the charge radius in the limit $M \to 0$ largely persists. While the form factor at high $Q^2$ has the right power law, the relative strength is found to be too small by a factor of the order $(M/(2\bar{e}))^4$. Both effects can be ascribed to the fact that the dependence on $Q^2$ appears only through the factor $v^2 = (Q^2/(4\,M^2 + Q^2))$ and involve interaction effects that make $M \neq (2\bar{e})$. This suggests that significant two-body currents are still missing.

Interestingly, many recent works looking at the frame dependence of form factors [26–28] show features similar to the above ones when only the valence contribution, excluding therefore two-body currents, is retained. These works were mainly performed in the light-front approach and, for some of them, have a field-theory foundation. The same features have also been observed in an instant-form calculation in the limit of an overall large momentum carried by the system under consideration [24]. In some cases, the form factors can be shown to depend on $Q$ through the ratio $Q/M$, as in point form results presented here. It is therefore legitimate to establish some relationship between the above works and the present one. For those of these studies that have a field-theory foundation, a complete analysis is possible showing what are the two-body currents that allow one to recover the "exact" result. It is thus found that these currents have a non-trivial structure, with an integrand behaving like 0/0 in the large momentum limit (instant-form case). Clearly, it seems that similar two-body currents are needed in the present case too. Having a non-perturbative character, they could not be obtained however from minimal conditions such as current conservation or reproducing the Born-amplitude behavior. Studying these currents and their implementation in the "point form" approach is an important task for the future. It would be in particular interesting to elucidate why they are needed in a formalism which ensures the frame independence of form factors, while they are required in other forms to restore this very property.

Present studies on the implementation of the "point form" approach have been devoted to a simple system. The ultimate aim however is to settle a formalism whose reliability is good enough to be applied to a physical system, the pion in first place. Apart from the necessity of introducing further two-body currents, as mentioned above, two aspects have to be considered: the spin-(1/2) nature of the constituents and the Goldstone character of the pion. The first one requires some care as the correct asymptotic behavior of the pion form factor is not expected to come from the one-body current [9] but from two-body currents that account for the contribution of extra components appearing in other formalisms [29]. As for the second one, it manifests itself by the low mass of the pion. As seen in this work, this is the case where the implementation of the "point form" approach is the worse. Being independent of the spin of the constituents, the effect requires a solution in the spin-less case. The present work, though concerned with an academic system, is therefore of the utmost importance.

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## Appendix A. Form factors in the Wick-Cutkosky model

This appendix contains expressions of form factors in the Wick-Cutkosky model which are used in the present work, successively for a scalar and a vector probe.

### Appendix A.1. Matrix element of a scalar current

– Matrix element of the current $n=1 \to n=1$:

$$\begin{aligned}F_0(q^2) &= \frac{i}{2}\int \frac{\mathrm{d}^4p}{4\pi}\,\mathrm{d}z_f\,\mathrm{d}z_i\\ &\times \frac{g_1(z_f)\,g_1(z_i)}{\left(m^2-(P_f^2-2P_f\cdot p)\,\frac{1+z_f}{2}-p^2-i\epsilon\right)^3}\\ &\times \frac{(p^2-m^2)}{\left(m^2-(P_i^2-2P_i\cdot p)\,\frac{1+z_i}{2}-p^2-i\epsilon\right)^3}\\ &= \frac{3\pi}{8}\int \mathrm{d}z_f\,\mathrm{d}z_i\,\mathrm{d}x\,\frac{g_1(z_f)\,g_1(z_i)\,x^2\,(1-x)^2}{D^4}\,I_0, \quad (\mathrm{A.1})\end{aligned}$$

with

$$\begin{aligned}I_0 &= \frac{1}{2}\left(m^2-\frac{1}{4}\,(Z_f+Z_i)\,\left(M_f^2\,Z_f+M_i^2\,Z_i\right)\right.\\ &\quad\left.-\frac{1}{4}\,Q^2\,Z_f\,Z_i\right)+\frac{1}{3}D,\\ D &= m^2-\frac{1}{4}\,(2-Z_f-Z_i)\,\left(M_f^2\,Z_f+M_i^2\,Z_i\right)\\ &\quad+\frac{1}{4}\,Q^2\,Z_f\,Z_i, \quad (\mathrm{A.2})\end{aligned}$$

where $Z_i=(1+z_i)\,x$ and $Z_f=(1+z_f)\,(1-x)$.

– Matrix element of the current $n=1,\ l=0 \to n=2,\ l=0$:

$$\begin{aligned}F_0(q^2) &= \frac{i}{2}\int \frac{\mathrm{d}^4p}{4\pi}\,\mathrm{d}z_f\,\mathrm{d}z_i\\ &\times \frac{g_2(z_f)\,g_1(z_i)}{\left(m^2-(P_f^2-2P_f\cdot p)\,\frac{1+z_f}{2}-p^2-i\epsilon\right)^4}\\ &\times \frac{\left((\frac{1}{2}P_f-p)^2+m^2-\frac{1}{4}P_f^2\right)(p^2-m^2)}{\left(m^2-(P_i^2-2P_i\cdot p)\,\frac{1+z_i}{2}-p^2-i\epsilon\right)^3}\\ &= \frac{\pi}{4}\int \mathrm{d}z_f\,\mathrm{d}z_i\,\mathrm{d}x\,\frac{g_2(z_f)\,g_1(z_i)\,(1-x)^3\,x^2}{D^5}\,I_0, \quad (\mathrm{A.3})\end{aligned}$$

with

$$\begin{aligned}I_0 &= \frac{1}{2}(2\,A\,B+D\,(C-A-B)+D^2),\\ H &= \frac{1}{4}\,\left(M_f^2(Z_f+Z_i/2)+M_i^2\,Z_i/2\right)+\frac{1}{8}\,Q^2\,Z_i,\\ A &= \frac{1}{4}\,(Z_f+Z_i)\,\left(M_f^2\,Z_f+M_i^2\,Z_i\right)+\frac{1}{4}\,Q^2\,Z_f\,Z_i-m^2,\\ B &= A-2\,H+2\,m^2, \quad C=-A+H-m^2. \quad (\mathrm{A.4})\end{aligned}$$

The quantity $D$, referred to here and below, is defined in eq. (A.2).

### Appendix A.2. Matrix element of a vector current

– Matrix element of the current $n=1 \to n=1$:

$$\begin{aligned}&F_1(q^2)\,(P_f^\mu+P_i^\mu)+F_2(q^2)\,q^\mu =\\ &i\int \frac{\mathrm{d}^4p}{4\pi}\,\mathrm{d}z_f\,\mathrm{d}z_i\,\frac{g_1(z_f)\,g_1(z_i)\,\left(P_f^\mu+P_i^\mu-2\,p^\mu\right)}{\left(m^2-(P_f^2-2P_f\cdot p)\,\frac{1+z_f}{2}-p^2-i\epsilon\right)^3}\\ &\times \frac{(p^2-m^2)}{\left(m^2-(P_i^2-2P_i\cdot p)\,\frac{1+z_i}{2}-p^2-i\epsilon\right)^3}\\ &= \frac{3\pi}{8}\int \mathrm{d}z_f\,\mathrm{d}z_i\,\mathrm{d}x\,\frac{g_1(z_f)\,g_1(z_i)\,(1-x)^2\,x^2}{D^4}\\ &\times\left(I_1\,(P_f^\mu+P_i^\mu)\,+\,I_2\,q^\mu\right), \quad (\mathrm{A.5})\end{aligned}$$

with $q^\mu = (P_f - P_i)^\mu$ and

$$I_1 = (2-Z_f-Z_i)\left(m^2-\frac{1}{4}\,M_f^2\,Z_f-\frac{1}{4}\,M_i^2\,Z_i\right)-\frac{1}{3}\,D,$$

$$I_2 = -\,(Z_f - Z_i)\left(m^2 - \frac{1}{4}\,M_f^2 Z_f - \frac{1}{4}\,M_i^2 Z_i\right). \quad \text{(A.6)}$$

– Matrix element of the current $n=1,\ l=0 \to n=2,\ l=0$:

$$F_1(q^2)\,(P_f^\mu + P_i^\mu) + F_2(q^2)\,q^\mu =$$

$$i\int \frac{\mathrm{d}^4p}{4\pi}\,\mathrm{d}z_f\,\mathrm{d}z_i \frac{g_2(z_f)\,g_1(z_i)\left(P_f^\mu + P_i^\mu - 2\,p^\mu\right)}{\left(m^2-(P_f^2-2P_f\cdot p)\,\frac{1+z_f}{2}-p^2-i\epsilon\right)^4}$$

$$\times\frac{\left((\frac{1}{2}P_f - p)^2 + m^2 - \frac{1}{4}P_f^2\right)(p^2-m^2)}{\left(m^2 - (P_i^2 - 2P_i\cdot p)\,\frac{1+z_i}{2} - p^2 - i\epsilon\right)^3}$$

$$= \frac{\pi}{4}\int \mathrm{d}z_f\,\mathrm{d}z_i\,\mathrm{d}x\,\frac{g_2(z_f)\,g_1(z_i)\,(1-x)^3\,x^2}{D^5}$$

$$\times\left(I_1\,(P_f^\mu + P_i^\mu) + I_2\,q^\mu\right), \quad \text{(A.7)}$$

with

$$I_1 = P_C + \frac{1}{2}\,P_A - \frac{1}{2}(Z_f+Z_i)\,(P_C+P_A+P_B),$$

$$I_2 = \frac{1}{2}\,P_A - \frac{1}{2}(Z_f - Z_i)\,(P_C + P_A + P_B),$$

$$P_A = \frac{1}{2}\,D\,(D-A), \qquad P_B = \frac{1}{2}\,D\,(D-B),$$

$$P_C = 2\,A\,B + D\,(C-A-B) + D^2. \quad \text{(A.8)}$$

where $A$, $B$ and $C$ are defined in eq. (A.4).

## Appendix B. Analytic expressions for form factors in the "point form" approach

Expressions of form factors calculated in the "point form" approach for a simple interaction model v0, first presented in ref. [17], are given together with details concerning their derivation as well as the non-relativistic expressions.

For the wave functions we use solutions obtained with a Coulomb potential,

$$\left[4\left(p^2+m^2\right)-M^2\right]\phi(\vec{p}) = -4m\int\frac{\mathrm{d}\vec{p}\,'}{(2\pi)^3}\,V_{\rm int}(\vec{p},\vec{p}\,')\,\phi(\vec{p}\,'),$$

$$\text{with}\quad V_{\rm int}(\vec{p},\vec{p}\,') = -\frac{g^2}{(\vec{p}-\vec{p}\,')^2}. \quad \text{(B.1)}$$

The wave functions of the ground and first-excited states, $\phi_i(\vec{p})$ and $\phi_f(\vec{p})$, respectively, are then given by

$$\phi_i(\vec{p}) = \sqrt{4\pi}\,\frac{4\,\kappa^{5/2}}{(\kappa^2+\vec{p}^{\,2})^2},$$

$$\phi_f(\vec{p}) = \sqrt{4\pi}\,\frac{8\kappa^{*5/2}}{(\kappa^{*2}+\vec{p}^{\,2})^3}\,(\vec{p}^{\,2}-\kappa^{*2}), \quad \text{(B.2)}$$

where $\kappa^2 = m^2 - \frac{1}{4}M^2$, $\kappa^{*2} = m^2 - \frac{1}{4}M^{*2}$, the total mass $M\,(M^*)$ being the one obtained from the Bethe-Salpeter equation ($\kappa^2 \simeq 4\,\kappa^{*2}$). The binding energy $E$, referred to in tables 1-6, is related to the total mass of a state by $M^2 = (2\,m - E)^2$. It has been shown that an equation like eq. (B.1) reproduces rather well the (normal) spectrum of the Wick-Cutkosky model, provided an effective coupling is used [31,32]. In particular, both models exhibit the same degeneracy pattern. The wave functions of eqs. (B.2) should therefore be a good zeroth-order approximation for our study, including for the extreme case $M^2 = 0$.

With these wave functions, some of the form factors can be calculated analytically. Thus, in the "point form" approach, the elastic form factor reads

$$F_1(q^2=-Q^2) = \frac{1+2\frac{Q^2}{4M^2}}{\left(1+\frac{Q^2}{4M^2}\right)^4\left(1+\frac{Q^2}{16\kappa^2\left(1+\frac{Q^2}{4M^2}\right)}\right)^2},$$

$$F_2(q^2=-Q^2) = 0. \quad \text{(B.3)}$$

Interestingly, the quantity $\frac{Q^2}{16\kappa^2}$ at the denominator of $F_1(q^2)$ is divided by the factor $1+\frac{Q^2}{4M^2}$. This one was introduced in many calculations to account for the Lorentz-contraction effect [10,11] but it turned out to be valid only at small $Q^2$. The inelastic form factors for a transition from the ground state to the first-excited radial state, $\tilde{F}_1(q^2)$ and $\tilde{F}_2(q^2)$ are given by

$$\tilde{F}_1(q^2) = (1+v^2)\,(1-v^2)^3 \times\frac{64\sqrt{2}\,\kappa^4\,v^2\,(16\,m^2-4\kappa^2(1-v^2))}{(9\kappa^2+v^2\,(16\,m^2-10\kappa^2)+v^4\kappa^2)^3},$$

$$\tilde{F}_2(q^2) = (1+v^2)\,(1-v^2)^4 \times\frac{64\,\sqrt{2}\,(3+v^2)\,\kappa^6}{(9\kappa^2+v^2\,(16\,m^2-10\kappa^2)+v^4\kappa^2)^3}, \quad \text{(B.4)}$$

where $v^2$ is defined after eq. (16). There is no known analytic expression for the form factor, $F_0(q^2=-Q^2)$.

In the non-relativistic case, the elastic and inelastic form factors are respectively given by

$$F_0(q^2) = F_1(q^2) = \frac{\kappa^4}{(\kappa^2+Q^2/16)^2}, \quad F_2(q^2) = 0,$$

$$F_0(q^2) = F_1(q^2) = \frac{64\,\sqrt{2}\,\kappa^4\,Q^2}{(9\kappa^2+Q^2)^3},\ F_2(q^2) = \frac{192\,\sqrt{2}\,\kappa^6}{(9\kappa^2+Q^2)^3}. \quad \text{(B.5)}$$

The equality $\kappa = 2\,\kappa^* = \frac{m\,\alpha_{\rm eff}}{2}$ has been assumed. Taking into account that $M_f^2 - M_i^2 = 3\,\kappa^2$, one can verify that the current conservation condition, eq. (2), is fulfilled.

To get analytic expressions for the form factors in the "point form" using Coulombian-type wave functions, the following relations have been employed:

$$\int {\rm d}\vec{p} \left(\kappa^2 + p_x^2 + p_y^2 + \left(\frac{p_z - v\,e_p}{\sqrt{1-v^2}}\right)^2\right)^{-1}$$
$$\times \left(\kappa^{*2} + p_x^2 + p_y^2 + \left(\frac{p_z + v\,e_p}{\sqrt{1-v^2}}\right)^2\right)^{-1} =$$
$$\pi^2 \frac{(1-v^2)^{3/2}}{v} \left[\frac{M+M^*}{M\,M^*} \arctan\left(\frac{v\,(M+M^*)}{2\,(\kappa+\kappa^*)}\right) - \frac{M-M^*}{M\,M^*} \arctan\left(\frac{v\,(M-M^*)}{2\,(\kappa+\kappa^*)}\right)\right],$$
$$\int {\rm d}\vec{p} \left(\kappa^2 + p_x^2 + p_y^2 + \left(\frac{p_z - v\,e_p}{\sqrt{1-v^2}}\right)^2\right)^{-1}$$
$$\times \left(\kappa^{*2} + p_x^2 + p_y^2 + \left(\frac{p_z + v\,e_p}{\sqrt{1-v^2}}\right)^2\right)^{-1} \frac{p_z}{e_p} =$$
$$\pi^2 \frac{(1-v^2)^{3/2}}{v^2} \left[\frac{M-M^*}{M\,M^*} \arctan\left(\frac{v\,(M+M^*)}{2\,(\kappa+\kappa^*)}\right) - \frac{M+M^*}{M\,M^*} \arctan\left(\frac{v\,(M-M^*)}{2\,(\kappa+\kappa^*)}\right)\right]. \quad \text{(B.6)}$$

Complete expressions for the form factors are obtained by adding appropriately derivatives of the above ones with respect to the quantities $\kappa^2$ or $\kappa^{*2}$, taking into account that $M$ and $M^*$ depend on them.

For the ground state, the elastic form factor in the "point form" can be directly obtained from the non-relativistic one by making a change of variable, starting from the above expression:

$$\int {\rm d}\vec{p} \left(\kappa^2 + p_x^2 + p_y^2 + \left(\frac{p_z - v\,e_p}{\sqrt{1-v^2}}\right)^2\right)^{-1}$$
$$\times \left(\kappa^2 + p_x^2 + p_y^2 + \left(\frac{p_z + v\,e_p}{\sqrt{1-v^2}}\right)^2\right)^{-1} =$$
$$(1-v^2)^{3/2} \int {\rm d}p_x\, {\rm d}p_y\, {\rm d}\tilde{p}_z\, \left[(\kappa^2 + p_x^2 + p_y^2 + v^2\,\tilde{m}^2)^2 + 2\tilde{p}_z^2(\kappa^2 + p_x^2 + p_y^2 - v^2\,\tilde{m}^2) + \tilde{p}_z^4\right]^{-1}. \quad \text{(B.7)}$$

The integrand is identical to the non-relativistic one, where $m^2$ has been replaced by $\tilde{m}^2 = m^2 - \kappa^2$ $(= M^2/4)$.

## Appendix C. Two-body currents and form factors in a Galilean approach

Expressions for the two-body currents in a relativized, Galilean-invariant approach (interaction model v1 in the text), together with their contributions to form factors are given in this appendix. Their presentation roughly follows the same structure as those for the "point form" case, eqs. (52), (53).

$$F_0(q^2) = \frac{\sqrt{N_f\,N_i}}{4\,m} \times \left(\int \frac{{\rm d}\vec{p}'\,{\rm d}\vec{p}}{(2\pi)^3}\,\phi_f(\vec{p}')\,\frac{2\,m}{e_p+e_{p'}}\,\phi_i(\vec{p})\;\delta\Big(\frac{1}{2}\vec{q}+\vec{p}'-\vec{p}\Big) + \int \frac{{\rm d}\vec{p}'\,{\rm d}\vec{p}}{(2\pi)^6}\,\phi_f(\vec{p}')\,\phi_i(\vec{p})\,\frac{m^2}{e_p e_{p'}}\,(K_{\Delta B})_0\right),$$

$$F_1(q^2) = \frac{\sqrt{N_f\,N_i}}{2\,\bar{M}} \times \left(\bar{F}\int \frac{{\rm d}\vec{p}'\,{\rm d}\vec{p}}{(2\pi)^3}\,\phi_f(\vec{p}')\,\phi_i(\vec{p})\;\delta\Big(\frac{1}{2}\vec{q}+\vec{p}'-\vec{p}\Big) + \bar{F}\int \frac{{\rm d}\vec{p}'\,{\rm d}\vec{p}}{(2\pi)^6}\,\phi_f(\vec{p}')\,\phi_i(\vec{p})\,\frac{m^2}{e_p e_{p'}}\,(K_{\rm int})_1 + \int \frac{{\rm d}\vec{p}'\,{\rm d}\vec{p}}{(2\pi)^6}\,\phi_f(\vec{p}')\,\phi_i(\vec{p})\,\frac{m^2}{e_p e_{p'}}\,(K_{\Delta B})_1\right),$$

$$F_2(q^2)\,\vec{q} = -\sqrt{N_f\,N_i} \times \left(\bar{F}\int \frac{{\rm d}\vec{p}'\,{\rm d}\vec{p}}{(2\pi)^3}\,\phi_f(\vec{p}')\,\frac{\vec{p}+\vec{p}'}{e_p+e_{p'}}\,\phi_i(\vec{p})\;\delta\Big(\frac{1}{2}\vec{q}+\vec{p}'-\vec{p}\Big) + \bar{F}\int \frac{{\rm d}\vec{p}'\,{\rm d}\vec{p}}{(2\pi)^6}\,\phi_f(\vec{p}')\,\phi_i(\vec{p})\,\frac{m^2}{e_p e_{p'}}\,(\vec{K}_{\rm int})_2 + \int \frac{{\rm d}\vec{p}'\,{\rm d}\vec{p}}{(2\pi)^6}\phi_f(\vec{p}')\,\phi_i(\vec{p})\,\frac{m^2}{e_p e_{p'}}\,(\vec{K}_{\Delta B})_2\right),$$

$$\text{with} \qquad \bar{F} = 1 - \left(\frac{\bar{M}-2\,\bar{e}}{\bar{M}+2\,\bar{e}}\right)^2, \quad \text{(C.1)}$$

and $\bar{M} = (M_i+M_f)/2$, $\bar{e} = (\bar{e}_i+\bar{e}_f)/2$. The normalization factors $N_{i,f}$ and the quantities $\bar{e}_{i,f}$ are defined by

$$\frac{1}{N_{i,f}} = \int \frac{{\rm d}\vec{p}}{(2\pi)^3}\,\phi_{i,f}^2(\vec{p})\,\frac{4\,e_p}{(M+2\,e_p)^2} = \frac{4\,\bar{e}_{i,f}}{(M+2\,\bar{e}_{i,f})^2} \int \frac{{\rm d}\vec{p}}{(2\pi)^3}\,\phi_{i,f}^2(\vec{p}). \quad \text{(C.2)}$$

The expressions of the $K$ quantities, which account for two-body currents, are given by

$$(K_{\rm int})_1 = 0,$$

$$(\vec{K}_{\rm int})_2 = -\frac{g^2}{\mu^2 + (\frac{1}{2}\vec{q} + \vec{p}\,' - \vec{p})^2} \frac{1}{e_{p'+q/2}\, e_{p-q/2}}$$
$$\times \left( e_p \frac{\vec{p}\,' + \vec{q}/4}{e_{p'+q/2} + e_{p'}} + e_{p'} \frac{\vec{p} - \vec{q}/4}{e_{p-q/2} + e_p} \right),$$

$$(K_{\Delta B})_0 = \frac{g^2}{\mu^2 + (\frac{1}{2}\vec{q} + \vec{p}\,' - \vec{p})^2}$$
$$\times \left( \frac{2\,m}{2\,e_{p'+q/2}\,(e_{p'+q/2} + e_p)} + \frac{2\,m}{2\,e_{p-q/2}\,(e_{p-q/2} + e_{p'})} \right),$$

$$(K_{\Delta B})_1 = \frac{g^2}{\mu^2 + (\frac{1}{2}\vec{q} + \vec{p}\,' - \vec{p})^2}$$
$$\times \left( \frac{e_{p'}\,(e_{p'+q/2} - e_{p'})}{2\,e_{p'+q/2}\,(e_{p'+q/2} + e_p)\,(e_{p'+q/2} + e_{p'})} \right.$$
$$\left. + \frac{e_p\,(e_{p-q/2} - e_p)}{2\,e_{p-q/2}\,(e_{p-q/2} + e_{p'})\,(e_{p-q/2} + e_p)} \right),$$

$$(\vec{K}_{\Delta B})_2 = \frac{g^2}{\mu^2 + (\frac{1}{2}\vec{q} + \vec{p}\,' - \vec{p})^2}$$
$$\times \left( \frac{e_{p'}\, q^0\, (\vec{p}\,' + \vec{q}/4)}{2\,e_{p'+q/2}\,(e_{p'+q/2} + e_p)\,(e_{p'+q/2} + e_{p'})^2} \right.$$
$$\left. + \frac{e_p\, q^0\, (-\vec{p} + \vec{q}/4)}{2\,e_{p-q/2}\,(e_{p-q/2} + e_{p'})\,(e_{p-q/2} + e_p)^2} \right), \tag{C.3}$$

where $q^0$ is defined as $M_f - M_i$, consistently with a Galilean-invariant calculation. While one could recover eqs. (37), (38) in some limit, the above expressions evidence significant differences. Equations (37), (38) contain denominators with four energy terms instead of two here. On the other hand, they do not contain a squared term at the denominator like in eq. (C.3). These differences illustrate the absence of a guide to derive two-body currents as soon as an effective interaction is used. Notice also that the term ensuring current conservation, $\vec{K}_{\rm int}$, does not contain any $1/q^2$ term.

## Appendix D. Two-body currents in the "point form" approach

The difference between the current in the full Born approximation and the one accounted for by solving eqs. (39), (40) is given by

$$J^{\mu}_{\Delta B}(q, p_{1i}, p_{2i}, p_{1f}, p_{2f}) =$$
$$-\sqrt{\frac{m}{\lambda_f \cdot p_{1f}} \frac{m}{\lambda_f \cdot p_{2f}}} \frac{g^2\,\delta(\vec{q} + \vec{P}_i - \vec{P}_f)}{\mu^2 - (p_{2i} - p_{2f})^2} \sqrt{\frac{m}{\lambda_i \cdot p_{1i}} \frac{m}{\lambda_i \cdot p_{2i}}}$$
$$\times \frac{2\lambda^{\mu}_i\,(\lambda_i \cdot p_{2i}) - p^{\mu}_{2f} + 2\lambda^{\mu}_f\,(\lambda_f \cdot p_{2f}) - p^{\mu}_{2f}}{4\,\lambda_i \cdot p_{2i}\,(\lambda_i \cdot p_{2i} - \lambda_i \cdot p_{2f})}$$
$$+ \sqrt{\frac{m}{\lambda_f \cdot p_{1f}} \frac{m}{\lambda_f \cdot p_{2f}}} \frac{g^2\,\delta(\vec{q} + \vec{P}_i - \vec{P}_f)}{\mu^2 - (p_{2i} - p_{2f})^2 + (\lambda_i \cdot (p_{2i} - p_{2f}))^2}$$
$$\times \sqrt{\frac{m}{\lambda_i \cdot p_{1i}} \frac{m}{\lambda_i \cdot p_{2i}}} \frac{2\lambda^{\mu}_i(\lambda^i \cdot p_{2f}) - p^{\mu}_{2f} + 2\lambda^{\mu}_f(\lambda_f \cdot p_{2f}) - p^{\mu}_{2f}}{4\,\lambda_i \cdot p_{2f}\,(\lambda_i \cdot p_{2i} - \lambda_i \cdot p_{2f})}$$
$$+ (i \leftrightarrow f). \tag{D.1}$$

This can be rewritten in a way which emphasizes the absence of the pole term $1/(\lambda_i \cdot p_{2i} - \lambda_i \cdot p_{2f})$:

$$J^{\mu}_{\Delta B}(q, p_{1i}, p_{2i}, p_{1f}, p_{2f}) =$$
$$\sqrt{\frac{m}{\lambda_f \cdot p_{1f}} \frac{m}{\lambda_f \cdot p_{2f}}} \frac{g^2}{2}\,\delta(\vec{q} + \vec{P}_i - \vec{P}_f) \sqrt{\frac{m}{\lambda_i \cdot p_{1i}} \frac{m}{\lambda_i \cdot p_{2i}}}$$
$$\times \left[ \left( \frac{\lambda^{\mu}_f(\lambda_f \cdot p_{2f}) - p^{\mu}_{2f}}{\lambda_i \cdot p_{2i}} + \lambda^{\mu}_i \right) \right.$$
$$\times \left( \frac{\lambda_i \cdot (p_{2f} - p_{2i})}{H(0)\,H(\lambda_i)} + \frac{(\lambda_i + \lambda_f) \cdot (p_{2i} - p_{2f})}{H(\lambda_f)\,H(\lambda_i)} \right)$$
$$+ \left( \frac{\lambda^{\mu}_i(\lambda_i \cdot p_{2i}) - p^{\mu}_{2i}}{\lambda_f \cdot p_{2f}} + \lambda^{\mu}_f \right)$$
$$\left. \times \left( \frac{\lambda_f \cdot (p_{2i} - p_{2f})}{H(0)\,H(\lambda_f)} + \frac{(\lambda_i + \lambda_f) \cdot (p_{2f} - p_{2i})}{H(\lambda_f)\,H(\lambda_i)} \right) \right]. \tag{D.2}$$

For the scalar probe, one gets similarly

$$S_{\Delta B}(q, p_{1i}, p_{2i}, p_{1f}, p_{2f}) =$$
$$-\sqrt{\frac{m}{\lambda_f \cdot p_{1f}} \frac{m}{\lambda_f \cdot p_{2f}}} \frac{g^2\,\delta(\vec{q} + \vec{P}_i - \vec{P}_f)}{\mu^2 - (p_{2i} - p_{2f})^2}$$
$$\times \frac{2\,m}{4\,\lambda_i \cdot p_{2i}\,(\lambda_i \cdot p_{2i} - \lambda_i \cdot p_{2f})} \sqrt{\frac{m}{\lambda_i \cdot p_{1i}} \frac{m}{\lambda_i \cdot p_{2i}}}$$
$$+ \sqrt{\frac{m}{\lambda_f \cdot p_{1f}} \frac{m}{\lambda_f \cdot p_{2f}}} \frac{g^2\,\delta(\vec{q} + \vec{P}_i - \vec{P}_f)}{\mu^2 - (p_{2i} - p_{2f})^2 + (\lambda_i \cdot (p_{2i} - p_{2f}))^2}$$
$$\times \frac{2\,m}{4\,\lambda_i \cdot p_{2f}\,(\lambda_i \cdot p_{2i} - \lambda_i \cdot p_{2f})} \sqrt{\frac{m}{\lambda_i \cdot p_{1i}} \frac{m}{\lambda_i \cdot p_{2i}}} + (i \leftrightarrow f)$$
$$= \sqrt{\frac{m}{\lambda_f \cdot p_{1f}} \frac{m}{\lambda_f \cdot p_{2f}}} \frac{g^2}{2}\,\delta(\vec{q} + \vec{P}_i - \vec{P}_f) \sqrt{\frac{m}{\lambda_i \cdot p_{1i}} \frac{m}{\lambda_i \cdot p_{2i}}}$$
$$\times \left[ \frac{1}{H(0)} \left( \frac{m}{\lambda_i \cdot p_{2i}\,\lambda_i \cdot p_{2f}} + \frac{m}{\lambda_f \cdot p_{2f}\,\lambda_f \cdot p_{2i}} \right) \right.$$
$$\left. - \left( \frac{m\,\lambda_i \cdot (p_{2i} - p_{2f})}{\lambda_i \cdot p_{2f}\,H(0)\,H(\lambda_i)} + \frac{m\,\lambda_f \cdot (p_{2f} - p_{2i})}{\lambda_f \cdot p_{2i}\,H(0)\,H(\lambda_f)} \right) \right]. \tag{D.3}$$

Contribution of a two-body current to the matrix element of the scalar or vector current:

$$\sqrt{2E_f\,2E_i}\,\langle f|\begin{pmatrix} S_{\Delta B} \\ J^\mu_{\Delta B}\end{pmatrix}|i\rangle =$$
$$\sqrt{N_f\,N_i}\frac{1}{(2\pi)^6}\int \mathrm{d}^4p_{1f}\,\mathrm{d}^4p_{1i}\,\mathrm{d}^4p_{2f}\,\mathrm{d}^4p_{2i}\,\mathrm{d}\eta_f\,\mathrm{d}\eta_i$$
$$\times\,\delta(p_{1f}^2-m^2)\,\delta(p_{1i}^2-m^2)\,\delta(p_{2f}^2-m^2)\,\delta(p_{2i}^2-m^2)$$
$$\times\,\theta(\lambda_f\cdot p_{1f})\,\theta(\lambda_f\cdot p_{2f})\,\theta(\lambda_i\cdot p_{1i})\,\theta(\lambda_i\cdot p_{2i})$$
$$\times\,\delta^4(p_{1f}+p_{2f}-\lambda_f\eta_f)\,\delta^4(p_{1i}+p_{2i}-\lambda_i\eta_i)$$
$$\times\,\phi_f\left(\left(\frac{p_{1f}-p_{2f}}{2}\right)^2\right)\,\phi_i\left(\left(\frac{p_{1i}-p_{2i}}{2}\right)^2\right)$$
$$\times\sqrt{(p_{1f}+p_{2f})^2\,(p_{1i}+p_{2i})^2}\,4\,m^2$$
$$\times\begin{pmatrix} \tilde{S}_{\Delta B} \\ \tilde{J}^\mu_{\Delta B}\end{pmatrix}(q,p_{1f},p_{2f},p_{1i},p_{2i}), \quad \text{(D.4)}$$

where $\tilde{S}_{\Delta B}$ ($\tilde{J}^\mu_{\Delta B}$) represents the quantity $S_{\Delta B}$ ($J^\mu_{\Delta B}$) of eqs. (D.2), (D.3) excluding the $\delta$-function and the normalization factors $m/e$ accounted for separately.

The corresponding one-body contribution (which differs from eq. (14) by the presence of the overall factor $\sqrt{N_f\,N_i}$ in place of $\sqrt{2\,M_f\,2\,M_i}$) reads

$$\sqrt{2E_f\,2E_i}\;\langle f|S|i\rangle = \frac{\sqrt{N_f\,N_i}}{(2\pi)^3}\int \mathrm{d}^4p\,\mathrm{d}^4p_f\,\mathrm{d}^4p_i\,\mathrm{d}\eta_f\,\mathrm{d}\eta_i$$
$$\times\,\delta(p^2-m^2)\,\delta(p_f^2-m^2)\,\delta(p_i^2-m^2)$$
$$\times\,\theta(\lambda_f\cdot p_f)\,\theta(\lambda_f\cdot p)\,\theta(\lambda_i\cdot p)\,\theta(\lambda_i\cdot p_i)$$
$$\times\,\delta^4(p_f+p-\lambda_f\eta_f)\,\delta^4(p_i+p-\lambda_i\eta_i)$$
$$\times\,\phi_f\left(\left(\frac{p_f-p}{2}\right)^2\right)\,\phi_i\left(\left(\frac{p_i-p}{2}\right)^2\right)$$
$$\times\sqrt{(p_f+p)^2\,(p_i+p)^2}\,2m\,. \quad \text{(D.5)}$$

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